Paracentesis modeling and VR-based interactive simulation with haptic display for clinical skill training and assessment

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Abstract - In this paper¹ we describe the development of an interactive virtual reality (VR) system that aims to realistically simulate specific paracentesis clinical procedures (particularly the procedure involved in the catheterization of the subclavian vein). Two elastostatic finite element (FE) models are developed to enable physically based simulation of the deformable tissues, particularly skin deflection during needle insertion. The first one is a two-dimensional approximation FE model, while the second one is a further simplified 1D model with inhomogeneous properties. Simulation results from both FE models are compared to real experimental data available in the literature (reporting on human skin deformation profiles) and demonstrate their applicability for realtime interactive applications under the specific constraints and assumptions of the paracentesis simulation application considered in this paper. Furthermore, a haptic feedback device is coupled with the VR-based simulation to provide the user with realistic feeling of the interaction forces applied during the simulated paracentesis procedure. The system described in this paper is developed in the frames of a research project aiming to develop a larger-scale virtual environment simulator of emergency room scenarios and protocols for clinical skill training and assessment.

I. INTRODUCTION

The barrier between theoretical knowledge and clinical performance is probably the biggest problem as far as education of medical doctors and nursing professionals is concerned. Efficient clinical training of healthcare professionals constitutes undoubtedly a very difficult task, which is nowadays primarily based on the close supervision and monitoring by a specialist trainer, on the patient's consent, but also on suitable general conditions; a combination of factors which is not always attainable since the main and primary concern always remains, that of treating the patient in the best possible way. Based on the above remarks, we can say that the difficulties associated with clinical training in specific medical practice hospital environments (such as, the Emergency Rooms or the Intensive Care Units, where the ultimate degree of dexterity together with "real-time" clinical skills are needed) are more than evident. The timely and persistent adaptation of the trainee medical doctor from the theoretical education field to the clinical "hands-on" practice on the hospital environment, constitutes undoubtedly a real challenge and a primary educational objective in the Health Sciences. Training on cadavers, which is still used today for over a century now, is not always adequate to cover other than special needs, not always of primary importance in surgical education and training. The use of animals constituted an "attractive" solution for decades, but for obvious reasons, related to human ethics as well as evolving legislation issues, as well as due to the need for important investments in terms of the related infrastructure, this solution has been practically abandoned. The experience obtained from the application of flight-simulation systems for the training of pilots, in an application environment that resembles in a way the field of clinical practice (in terms of the need for rapid complex data evaluation and critical decisionmaking) has more recently increased interest towards the use of Virtual Reality (VR) technologies in the medical education field.

In this paper we describe on-going work that is carried out in the frames of a research project aiming to develop a computer-assisted educational platform based on a dynamic simulation system of an emergency room (ER) hospital environment. A set of clinical protocols and real case scenarios is implemented within the ER simulator, also incorporating VR techniques for visualization and user interaction. The system will enable the user to examine (virtual clinical data examination), evaluate (vital signs, clinical findings and medical images, obtained from a database of pre-stored clinical cases) and treat the virtual patient (for instance, by commanding the execution of certain clinical procedures like medicine supply). The system is designed to support both asynchronous self-education and training in specific

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clinical protocols, as well as real-time collective training through the dynamic simulation of realistic clinical scenarios, involving active collaboration of two or more actors (including medical/nursing trainees and trainers) under time pressure.

In this context, we are developing an interactive virtual reality system that aims to realistically simulate specific paracentesis clinical procedures, and more particularly the procedure involved in the catheterisation of the subclavian vein. This is one of the most common procedures in clinical practice and is often involved in the treatment of patients in an ER situation. This operation is particularly difficult to learn, and requires a combination of visual and haptic skills in order to identify the needle insertion point, and to control the needle position and orientation during penetration. To develop a realistic and accurate physically based dynamic simulation, two elastostatic finite element (FE) models are developed for the deformable tissues and anatomical structures of the human body involved in this procedure (particularly skin deflection during needle insertion). The first one of these models is a two-dimensional approximation FE model, while the second one is a further simplified 1D model with inhomogeneous properties. Simulation results from both models are compared to real experimental data available in the literature (reporting on human skin deformation profiles) and demonstrate their applicability for real-time interactive applications under the specific constraints and assumptions of the paracentesis simulation application considered in this paper.

Moreover, to incorporate the sense of touch and improve the impact of the training system in terms of "teaching basic clinical skills", a haptic interaction system is currently under development, coupling the VR-based simulation with a haptic device (a PhantomTM desktop force feedback device). The challenges addressed at this stage are twofold: (i) from a technical point of view, to achieve a trade-off between realism (that is, accuracy of the physically based dynamic simulation) and real-time performance (necessary particularly for a stable and haptic interaction with the system), "transparent" imposing model simplifications, based on specific assumptions and approximations, in order to achieve fast computations for the feedback forces distribution; (ii) from an educational/training point of view, to conduct human factors evaluation studies in order to identify the critical factors affecting the performance of the system in terms of clinical skill training and assessment.

II. TISSUE DEFORMATION MODELING FOR PARACENTESIS SIMULATION

The first step in achieving a realistic and accurate physically based paracentesis simulation is to develop models for the deformable tissues and anatomical structures of the human body involved in this clinical procedure. Elastic deformation modelling of soft tissues is a wide research area in the field of 3D computer graphics and animation, with many applications particularly in VR-based systems. The two most widely employed generic methodologies are based on: (a) massspring models, where the deformable object is modelled as a lattice of point masses interconnected by spring/damping elements; and (b) finite element (FE) models, which are based on the discretisation of a continuum elasticity model [Gibson et al., 1997].

Particularly, modelling the deformation of human tissues and organs for surgery simulation has turned out to be a real challenge, due to the lack of accurate knowledge about their physical properties, and the very few existing experimental data provided by in-vivo studies. The need is for both realism and speed, but for surgical simulation systems the emphasis is definitely put on real-time interaction. This has turned scientific interest towards the development of accurate mathematical models, but which can be at the same time treated using efficient numerical computation algorithms. Examples of such methodologies for modelling human soft tissue by applying FE techniques in a computationally efficient framework are presented in [Bro-Nielsen, 1998] and [Cotin et al., 1999]. More recently, the use of FE models to develop a needle insertion simulation system is proposed in [DiMaio et al., 2003].

The use of FE models is typically preferred for interactive surgical simulation applications, mainly because these models are parameterised and tuned more intuitively than mass-spring nets. However, to enable real-time interaction with 3D FE models, long pre-processing steps are required, imposing additional constraints related to shape changes and deformation limits. The use of simplifications" for particular "intelligent model applications may overcome these problems in a taskspecific context. In this framework, we propose the use of simplified 2D and 1D finite element models to simulate deformation of the skin, based on an assumption of radial symmetry properties in specific configurations of needle insertion operations. Our goal is to achieve a realistic interaction, both visual (based on deformation rendering) and haptic (based on real-time reflected force computation), in real-time and with a "reasonable" accuracy. This means that visual and haptic results should match as closely as possible available real experimental data provided by in vivo studies reported in the literature.

A. Two-dimensional elastostatic model

In this work we study the implementation of simplified (i.e. reduced dimension) FE models to simulate skin deformation in paracentesis operations. Assuming radial symmetry around the needle-skin contact point, we start the analysis by considering a two-dimensional (2D) model to approximate real three-dimensional

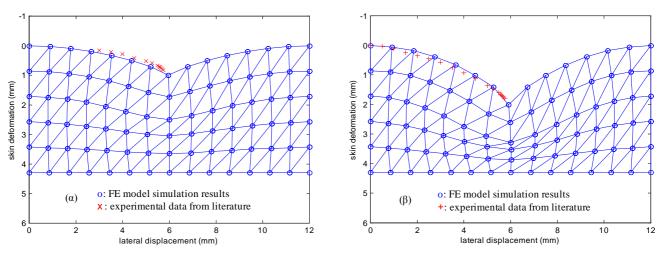


Fig.1. Skin deformation predicted by the 2D FE model (1mm and 2mm vertical deflection) compared with data from [Srinivasan, 1989]

deformation. The model is constructed based on triangular simplex elements. For the purposes of deformation modeling and simulation, we ignore dynamic viscoelastic behavior of the tissue, and consider the static elasticity problem. Following classic FEM formulation (for instance, see [Fagan, 1996]), the elastostatic deformation problem then reduces to the solution of a linear (sparse) matrix equation system:

$$\mathbf{K} \cdot \underline{U} = \underline{f} \tag{1}$$

where **K** is the global stiffness matrix, \underline{U} is the vector containing deformations on the nodes of the FE model, and \underline{f} are external forces acting on the system.

Boundary conditions are imposed on the system by fixing a set of nodes to pre-specified deformation level (e.g. $U_k=0$, for certain node k), and thus reducing the effective degrees of freedom (dof) of the system. For instance, for a 2D FE model with n_i nodes in one direction and n_j nodes in the other direction, the system possesses a total of:

 $n_{\text{dof}} = 2 \times n_i \times n_j$ degrees of freedom, for a total of $n_e = 2 \times (n_i - 1) \times (n_j - 1)$ simplex (triangular) elements

The global stiffness matrix is constructed from the "assembly" of all the individual element stiffness matrices \mathbf{K}^{e} (*e*=1,..., *n*_e), which are given by:

$$\mathbf{K}^{e} = \begin{bmatrix} \mathbf{B}^{e} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \mathbf{D}^{e} \end{bmatrix} \begin{bmatrix} \mathbf{B}^{e} \end{bmatrix} \cdot tA$$
(2)

where *t* is the constant thickness of the model, and *A* is the cross-sectional area of each element. For each element *e* of the model (*e*=1,...,*n*_e), [**B**^{*e*}] is a 3×6 matrix that results from the differentiation of the shape functions $N^{e}(x,y)$. If $\underline{\boldsymbol{u}}(x,y)=[\boldsymbol{u},\boldsymbol{v}]^{T}$ is the deformation vector of the 2D elastic continuum at point (*x*,*y*) within element *e*, we have:

$$\underline{\boldsymbol{u}}(x,y) = \begin{bmatrix} u \\ v \end{bmatrix} = \mathbf{N}^{e}(x,y) \cdot \underline{\boldsymbol{U}}^{e} =$$

$$= \begin{bmatrix} N_{1} & 0 & N_{2} & 0 & N_{3} & 0 \\ 0 & N_{1} & 0 & N_{2} & 0 & N_{3} \end{bmatrix}^{(e)} \cdot \begin{bmatrix} U_{1} \\ U_{2} \\ U_{3} \\ U_{4} \\ U_{5} \\ U_{6} \end{bmatrix}^{(e)}$$
(3)

where \underline{U}^e is the 6×1 vector containing the (2 dof) deformations for each one of the 3 nodes of the triangular element. The general form of the linear interpolation shape functions is given by (for k=1,...,3):

$$N_{k}(x, y) = (a_{k} + b_{k}x + c_{k}y)/(2A)$$
(4)

For simplex (triangular) elements, the parameters a_k , b_k , c_k (k=1,...,3) are then given by:

$$\begin{cases} a_1 = x_2 y_3 - x_3 y_2 \\ a_2 = x_3 y_1 - x_1 y_3 \\ a_3 = x_1 y_2 - x_2 y_1 \end{cases}, \begin{cases} b_1 = y_2 - y_3 \\ b_2 = y_3 - y_1 \\ b_3 = y_1 - y_2 \end{cases}, \begin{cases} c_1 = x_3 - x_2 \\ c_2 = x_1 - x_3 \\ c_3 = x_2 - x_1 \end{cases}$$
(5)

where x_k , y_k are the x-y coordinates of the k^{th} element's node. For each element *e* of the FE model, the strain ε is then approximated by:

$$\underline{\varepsilon} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \end{bmatrix} = \begin{bmatrix} \mathbf{B}^e \end{bmatrix} \cdot \underline{U}^e \tag{6}$$

The $[\mathbf{B}^e]$ matrix is of size 3×6, and it is thus found in this case to be composed only of the following constants:

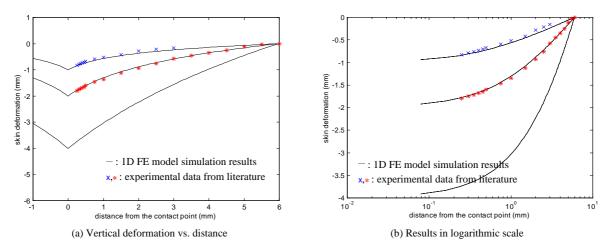


Fig. 2. Skin deformation predicted by the 1D FE model compared with experimental data from [Srinivasan, 1989].

$$\begin{bmatrix} \mathbf{B}^{e} \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} b_{1} & 0 & b_{2} & 0 & b_{3} & 0 \\ 0 & c_{1} & 0 & c_{2} & 0 & c_{3} \\ c_{1} & b_{1} & c_{2} & b_{2} & c_{3} & b_{3} \end{bmatrix}$$
(7)

Regarding now the material property matrix $[\mathbf{D}^e]$ of each element *e*, we have (using the plane stress type):

$$\begin{bmatrix} \mathbf{D}^{e} \end{bmatrix} = \frac{E}{1+\nu^{2}} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu/2) \end{bmatrix}$$
(8)

where E is the elasticity (Young's) modulus and v the Poisson's ratio of the material.

We applied this model to the problem of skin deformation modeling in paracentesis simulation. In this case, that is, we assume a *point load* applied to a specific (boundary) node k_l of a 2D FE structure. The model is considered to be both homogeneous and isotropic. The main issue here is to develop a model that will give in real-time "realistic" results under such loads. This can be assessed by comparing the obtained simulation results with real experimental data available in the literature, reporting on human skin surface deformation under various types of external loading. Such data are for instance reported in [Srinivasan, 1989], providing in vivo measurements of skin surface deflection profiles under line loads.

The goal of this study is to assess up to which extent a 2D FE model can provide adequate approximation for the considered application. We considered a multi-layered model consisting of n_y layers in the -y (vertical deformation) direction, and n_x elements in the -x (lateral displacement) direction. We conducted a preliminary parametric study, to identify the optimal number of layers and the ratio of the elastic moduli between the layers. We found that a 2D FE model consisting of 5 layers with

interlayer elasticity ratio of: {10-10-1-1-1} (i.e. the first two layers, epidermis-dermis, having an elasticity modulus that is 10 times the modulus of the three subcutaneous layers) can adequately approximate the elastic behavior of human skin under point load. It must be noted here that absolute value of the E modulus of the first layer, and consequently of the next layers, is not important for obtaining the deformation profile of the model, since no force calculation takes place at this stage; only relative ratio of the E moduli between successive layers is of importance at this stage. Actual numerical values of the elasticity modulus for the first two layers of the model (in-vivo human dermis) are taken according to the experimental data and the analysis presented in [Dandekar et al., 2003]: E≈0.2MPa. For the Poisson's ratio v we choose a value close to 0.5 (e.g. v=0.48) to incorporate the incompressibility properties of human tissue in the model.

Simulation results from application of this model are illustrated in Figure 1, where a total of as little as 14x5x2=140 elements was used, which is particularly important if we want to apply this model in real-time interactive systems. These results are compared to real experimental data reported in [Srinivasan, 1989], and demonstrate that for different vertical skin deflection amplitudes (1mm and 2mm), even this simple 2D FE model can adequately approximate the deformation profiles of the human skin under such point loads. Our findings are also consistent with results reported in [Dandekar et al., 2003], who proposed a five-layers 3D FE model for the fingerpad (consisting however of at least 8,500 nodes up to 29,000 in the high-resolution case).

B. One-dimensional simplified FE model

In this paragraph, we further pursue our study in the direction of investigating FE models simplifications, for real-time interactive soft tissue simulation under specific

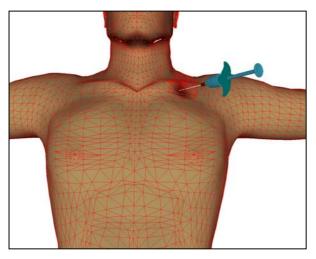


Fig.3. Collision detection and selection of triangles to deform during simulated needle insertion operation

application constraints and assumptions (such as, vertical point contact load, with radial symmetry assumption). In this application context, we developed a one-dimensional elastostatic model to simulate skin deformation in paracentesis operations. This second model consists of a simplified 1D "chord-like" medium, exhibiting inhomogeneous properties both in deformation magnitude and in lateral distance from the contact point. The media is again discretised using the FE method [Fagan, 1996], and is considered to be sub-divided into separate homogeneous (constant-stiffness) regions. The model is parameterised according to an assumed linear stiffness increase away from the center (considered as the contact, needle insertion point), also assuming linearly increasing constant-stiffness region width. Besides that, we also assume inhomogeneous characteristics in depth, with stiffness coefficients increasing with the deformation magnitude *u*, according to a typical exponential formula:

$$K = K_0 \Big[e^{m(u-u_0)} - 1 \Big] + K_{\min} \quad (u > u_0)$$
(9)

This term is used to model the instantaneous elastic component of force response for soft tissues exhibiting viscoelastic behavior (such as human skin) under compressive loads. u_0 is the value of small deformation below which (i.e. when $u \le u_0$) stiffness is experimentally observed to have a minimum constant value K_{\min} . K_0 and *m* are constants that can be determined from available experimental data. Following the experimental analysis reported in [Pawluk & Howe, 1999] (for human fingerpad deformation), we have set parameter m to a mean value of 2mm⁻¹, with K_0 =0.01N/mm. This gives a force response of approximately 5grams for 1mm indentation. These values should increase empirically by a factor of approximately 2 and 4 for line and flat plate loading, respectively. These results are also consistent with data presented in [Dandekar et al., 2003].

Simulation results are shown in Figure 2 (number of elements = 150), where vertical deformation is plotted against distance from the contact point. The obtained results are compared with the experimental data reported in [Srinivasan, 1989], providing in vivo measurements of skin surface deflection profiles under line loads. We can see that the simplified 1D finite element model, with inhomogeneous properties as described above, accurately predicts the real skin deformation profiles. Therefore, it can be used, even at this simple form, to provide a realistic and computationally efficient simulation for the specific paracentesis application context considered in this paper.

III. VIRTUAL REALITY SYSTEM: VISUAL AND HAPTIC DISPLAY

A. 3D Graphical Models and Visual Display

The FE-based skin-deformation simulation method described in the previous section is integrated within the VR-based paracentesis simulator that is currently under development. The 3D (surface) models of the human body (outer skin), as well as those for the different anatomical structures (inner organs) involved in such clinical operations (venal tree, muscle groups, lungs and bony structures), are extracted from 3D anatomical models that are commercially available (for the time being. we models available use at http://www.cacheforce.com/). These models are linked to the main application program, which is implemented using C++ and OpenGL for 3D rendering.

An important issue here is real-time collision detection between moving polyhedral objects in the virtual scene. In the first prototype platform presented in this paper, we have used the "ColDet" 3D collision detection library for generic polyhedra, which is freely available in the net: (http://photoneffect.com/coldet/). In our application, the collision area between the moving syringe and the human anatomical structures (skin etc.) is detected, and then used to identify the triangles of the polyhedral mesh of the human skin that are to be deformed (see Fig.3). Moreover, "dynamic remeshing" is to be performed "around" the contact (needle insertion) point, to increase locally the resolution of the triangular mesh, and enable a more realistic visual display of the skin deformation profile. This algorithm is currently in the stage of integration and testing, and will be reported in the near future.

Figure 4 shows two screen snapshots of the VR-based paracentesis simulation, including 3D visualisation of deformed skin (in solid and alpha-transparency mode) and venal tree, during simulated needle insertion. In the current simulator configuration, the system continuously monitors and registers the trajectory of the needle, and its intersection with the surface of the skin and the

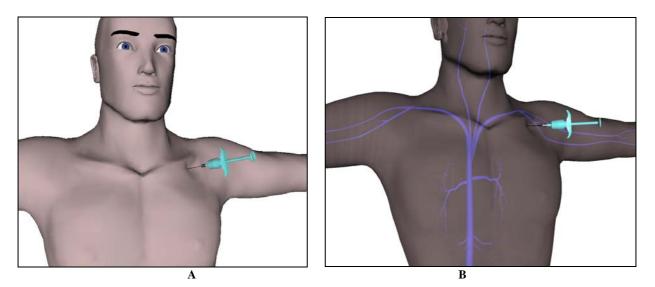


Fig. 4. VR-based paracentesis simulation: 3D visualisation in solid and alpha-transparency mode.

subclavian vein. The goal is twofold: (a) to provide realistic visual (skin deformation) and haptic (computation of reflected force) feedback, and (b) to enable the development of computer-automated skill assessment modules for different clinical training scenarios, as described in the following sections.

B. Haptic Display

Learning to safely and accurately perform a specific paracentesis operation is a particularly difficult task, requiring a combination of visual and tactile skills to identify the needle insertion point, and control the needle position and orientation during penetration. To provide realistic feedback to the medical/nursing trainee or novice user and improve the impact of the training system in terms of "teaching basic clinical skills", the system should not be limited in providing only 3D visualizations of the simulated medical procedures, but must also enable real-time haptic interaction, incorporating the sense of touch in the dynamic simulation. For this reason, the VRbased simulation is coupled with a haptic device (a PhantomTM Desktop force feedback device, from SensAble Technologies: http://www.sensable.com/), in order to provide the user with realistic feeling of the interaction forces applied during the simulated paracentesis procedure (Fig.5).

The haptic interaction system in a paracentesis simulation platform, should ideally involve both hands of the user, working in coordination to perform the simulated clinical procedure. For instance, when inserting a needle during paracentesis of the subclavian vein, the left hand palpates the clavicle bone anatomic area to find and recognize the anatomic landmark used as guide point, while the right hand inserts and orientates the needle through the selected point on the skin. Such dual-hand coordinated actions should be supported by the final platform. This is a challenging objective, both from technical and "humanfactors" perspective, and constitutes on its own an innovative aspect of the system that is currently under development, particularly concerning integration of such a "haptic interaction" mode into a complete clinical dexterity enhancement and skill assessment platform.

C. Clinical Skill Training and Assessment Platform

The work presented in this paper is performed within the frames of a national research project named "VRES: Virtual Reality Environment for Training and Assessment of Clinical Skills". Our final goal is to integrate this interactive simulator within a larger-scale computer-assisted clinical education and training platform, based on the dynamic simulation of complete clinical scenarios and medical protocols. The initial focus of the VRES platform is targeted onto a set of medical/nursing procedures within a simulated emergency room (ER) environment. Initial scenarios are being developed around a complete set of cardio-pulmonary resuscitation (CPR) protocols in ER/trauma cases. From a technical point of view, the key issues here concern: (i) modelling and real-time



Fig.5. Haptic interaction during paracentesis simulation with the Phantom[™] Desktop force-feedback device

computer-based simulation of selected clinical scenarios, built upon a multi-parameter decision-tree model, (ii) definition and implementation of training scenarios, with the emphasis being on real-time dynamic simulation and user interaction with the system, (iii) definition and implementation of the appropriate user interaction metaphors, including VR navigation, and multimodal (visual/haptic) feedback.

Paracentesis of specific deep vessels (such as the subclavian vein) is one of the most common procedures used in clinical practice and is often involved in the treatment of patients in an ER environment. This invasive procedure is particularly difficult to learn, and "hands-on" training may eventually include substantial risk for the patient. The VR-based haptic simulator proposed in this paper, when completed and operational, aims to help clinicians acquire some basic skill prior to any real clinical operation. For this purpose, we are also focusing our efforts on developing a module that will perform computer-assisted clinical skill evaluation and assessment, for a set of paracentesis training scenarios. For each training scenario, a number of appropriate scores will be computed based on a set of established performance indices. For instance, using the interactive simulation features of the system presented in the previous sections, we can measure on-line a set of dynamic parameters, such as the amount of tissue deformation (related to human tissue damage), when performing the simulated paracentesis operation, as well as the "optimality" of the path followed by the needle during skin puncturing and insertion into a deep vessel. Based on such measures, average or special-purpose performance scores can then be established, while the system can also advice the user on how to ameliorate his/her technique.

IV. CONCLUSION AND FUTURE WORK

This paper described the development of an interactive virtual reality (VR) system aiming to realistically simulate specific paracentesis clinical procedures (particularly catheterization of the subclavian vein). Simplified 1D and 2D finite element models are used for the physically based simulation of tissue deformation. The proposed models, simulating skin deflection during needle insertion, are experimentally validated compared to real skin deformation data available in the literature. In the future, a more complex layered structure will be considered, involving 3D anatomical models not only of the skin and subcutaneous fat layers, but also of underlying structures (major muscle groups, lungs, and bones).

The principal objective of this research is to enable medical/nursing students and trainees to acquire basic clinical skills through unlimited practicing in VR, before even touching a real patient, and thus without invoking any pain, discomfort or risk for real patients. However, our goal is to develop a system that will also provide selfevaluation and skill-assessment functionalities, as well as automatic correction and computer-assisted guidance features, through the detailed analysis of the technical mistakes performed by the trainee. It is thus foreseen that the same real-time haptic interaction platform will not only be used as a tool for dexterity enhancement, but also to monitor the performance of the trainee, introducing objective quantifiable measures and standardized criteria for the evaluation of clinical adequacy, technical dexterity, and psychokinetic skills. Our final goal is to integrate this interactive simulator within a larger-scale computer-assisted clinical education/training platform, based on the interactive simulation of complete scenarios and protocols related to the operation in an emergency room environment, for clinical skill training and assessment.

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