

# Answering Queries Determined by Views

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## Paper 188

December 9, 2005

### Abstract

Answering queries using views is the problem which examines how to derive the answers to a query when we only have the answers to a set of views. In this paper we investigate this problem in the case where the answers to the views uniquely determine the answers to the query. We say that a view set  $\mathcal{V}$  *determines* a query  $Q$  if for any two databases  $D_1, D_2$  it holds:  $\mathcal{V}(D_1) = \mathcal{V}(D_2)$  implies  $Q(D_1) = Q(D_2)$ . We consider the case where query and views are defined by conjunctive queries. We ask the question: If a view set  $\mathcal{V}$  determines a query  $Q$ , is there an equivalent rewriting of  $Q$  using  $\mathcal{V}$ ? Clearly if we can find an equivalent rewriting then the complexity of answering the queries given a view instance is polynomial. In this paper we show that computing the answers to queries determined by views is in  $NP \cap coNP$ . We find cases, (such as chain queries, views without nondistinguished variables) where if a query is determined by a view set then there is an equivalent rewriting, hence the complexity of answering the query on a view instance is polynomial. We reduce the general problem to special cases (such as boolean queries, binary base predicates). We introduce a problem which is a special case of the general problem and relates determinacy to query equivalence.

### 1 Introduction

The problem of using materialized views to answer queries [LMSS95] has received considerable attention because of its relevance to many data-management applications, such as information integration [B<sup>+</sup>97, C<sup>+</sup>94, HKWY97, IFF<sup>+</sup>99, LRO96, Ull97], data warehousing [TS97],[ACN00] web-site designs [FLSY99], and query optimization [CKPS95]. The problem can be stated as follows: given a query  $Q$  on a database schema and a set of views  $\mathcal{V}$  over the same schema, can we answer the query using only the answers to the views, i.e., for any

database  $D$ , can we find  $Q(D)$  if we only know  $\mathcal{V}(D)$ ?

A related fundamental question has recently arisen which is related to the information that is provided by a set of views for a specific query [SV05]. Thus, we say that a set  $\mathcal{V}$  of views *determines* a query  $Q$  if for any two databases  $D_1, D_2$  it holds:  $\mathcal{V}(D_1) = \mathcal{V}(D_2)$  implies  $Q(D_1) = Q(D_2)$ . A database query  $Q$  can be thought of as defining a partition of the set of all databases in the sense that databases on which the query produces the same set of tuples in the answer belong to the same equivalence class. In the same sense a set of views defines a partition of the set of all databases. Thus, if a view set  $\mathcal{V}$  determines a query  $Q$ , then the views' partition is a refinement of the partition defined by the query. Thus if we are given  $\mathcal{V}(D)$  only, then we can "find"  $Q(D)$  by noting the equivalence class of  $\mathcal{V}(D)$  and seeing what  $Q$  computes on that equivalence class. However, it is not easy to see whether the mapping from the views' equivalence class to the query equivalence class is even computable. We show in this paper that if a CQ view set determines a CQ query then computing the answers to the query is in  $NP \cap coNP$ .

A large amount of work in answering queries using views concerns finding rewritings of queries using a set of views. When there is an equivalent rewriting of a query  $Q$  using a set of views  $\mathcal{V}$  then  $\mathcal{V}$  determines  $Q$ . How about the converse? Given that  $\mathcal{V}$  determines  $Q$ , can we say that there exists an equivalent rewriting of  $Q$  using  $\mathcal{V}$ ? The existence of rewritings depend on the language of the rewriting and the language of the query and views. Given query languages  $\mathcal{L}, \mathcal{L}_{\mathcal{V}}, \mathcal{L}_Q$  we say that a language  $\mathcal{L}$  is *complete* for  $\mathcal{L}_{\mathcal{V}}$ -to- $\mathcal{L}_Q$  rewritings if whenever a set of views  $\mathcal{V}$  in  $\mathcal{L}_{\mathcal{V}}$  determines a query  $Q$  in  $\mathcal{L}_Q$  then there is a rewriting of  $Q$  in  $\mathcal{L}$  which uses only  $\mathcal{V}$ . In [SV05] this problem is investigated and is shown that there are cases where a certain language is not complete, e.g., it is shown that the language of union of conjunctive queries (UCQ) is not complete for UCQ-

to-UCQ rewritings. However it is noticed that a hard case to settle and hence an open problem is whether the language of conjunctive queries (CQ) is complete for CQ-to-CQ rewritings. In this paper we answer this question positively in special cases and also we show that some special cases are as hard to resolve as the general problem by reducing the general problem to them. Finally we introduce a seemingly simpler problem that relates determinacy and query equivalence which also remains open, whereas here we solve a special case of it.

The organization and the contribution of the paper are as follows: In Section 3 we show that if the views determine the query then the query answering problem is in  $NP \cap coNP$ . In Section 4 we prove that there is an algorithm to find an equivalent rewriting in case there exists one. In particular, we show that if there is an equivalent rewriting then the *canonical rewriting* is such a rewriting. We show how to construct a canonical rewriting. In Section 5 we present three special cases of CQ-to-CQ rewritings for which CQ is complete. The special cases are: when views have no nondistinguished variables, when views and query are chain queries and when the query has a single variable and the view set contains a single view with one nondistinguished variable. Hence, in these special cases the query answering problem is in PTIME. A summary of all CQ cases for which we know that CQ is complete is in Table 1.

In Section 6 we ask the question: If a single view determines the query then are there some natural conditions to add so that the query and view are equivalent? We identify such conditions and show that if they hold for a view and query then the following is true: If CQ is complete for CQ-to-CQ rewritings, and the view determines the query, then the view and query are equivalent. Thus we have a new variant of the problem which, although it seems like an "easier" problem to solve, this also remains open; here we solve a special case of it. In Section 7 we reduce the original problem to a special case where, if an equivalent rewriting exists then it is a projection of a single view. In Section 8 we present more reductions of the general problem to special cases, such as when we use only binary base predicates and when the query is Boolean. We also include a reduction which addresses connectivity issues related to determinacy.

**1.1 Related Work** In [SV05], the problem of determinacy is investigated for many languages including first order logic and fragments of second order logic and a considerable number of cases are resolved. The results closer to our setting show that if a language  $\mathcal{L}$  is complete of UCQ-to-UCQ (i.e., unions of CQs) rewritings, then  $\mathcal{L}$  must express non-monotonic queries. More-

over, this holds even if the database relations, views and query are restricted to be unary. This says that even Datalog is not complete for UCQ-to-UCQ rewritings. Datalog is not complete even for CQ<sup>≠</sup>-to-CQ rewritings. For CQ query and views, it is shown in [SV05] that CQ is complete for CQ-to-CQ rewritings iff whenever a set of views  $\mathcal{V}$  determines a query  $Q$  over finite instances then  $\mathcal{V}$  determines  $Q$  over unrestricted (i.e., may be infinite too) instances. They also prove that for unary or Boolean CQ views, then CQ is complete for CQ-to-CQ rewritings. For the unrestricted case, they prove that CQ is complete for CQ-to-CQ rewritings, however no monotonic language is complete for UCQ-to-UCQ rewritings.

Determinacy and notions related to it are also investigated in [GT00] where the notion of subsumption is introduced and used to the definition of complete rewritings and in [CdGLV02] where the concept of lossless view with respect to a query is introduced and investigated both under the sound view assumption (a.k.a. open world assumption) and under the exact view assumption (a.k.a. closed world assumption) on regular path queries used for semi-structured data. Losslessness under the CWA is identical to determinacy. There is a large amount of work on equivalent rewritings of queries using views. It includes [LMSS95] where it is proven that it is NP-complete whether a given CQ query has an equivalent rewriting using a given set of CQ views, [CR97] where polynomial subcases were identified. In [RSU95], [ALM02], [DG97] cases were investigated for CQ queries and views with binding patterns, arithmetic comparisons and recursion, respectively. In some of these works also the problem of maximally contained rewritings is considered. Intuitively, maximally contained rewritings is the best we can do when there is no equivalent rewriting and want to obtain a query that uses only the views and computes as many certain answers [AD98] as possible. In [LBU01] the notion of p-containment and equipotence is introduced to characterize view sets that can answer the same set of queries. Answering queries using views in semi-structured databases is considered in [CdGLV02] and references therein.

## 2 Preliminaries

**2.1 Basic Definitions** We consider queries and views defined by conjunctive queries (CQ for short) (i.e., select-project-join queries) in the form:

$$h(\bar{X}) : -g_1(\bar{X}_1), \dots, g_k(\bar{X}_k).$$

Each subgoal in the *body* is a *relational atom*. In each subgoal  $g_i(\bar{X}_i)$ , predicate  $g_i$  defines a *base relation* (we use the same symbol for the predicate and the relation), and every argument in the subgoal is either a variable or a constant. A variable is called *distinguished* if it

Query	Views	Reference
any	without nondistinguished	this paper
any	unary	[SV05]
any	boolean	[SV05]
single variable	single view, binary, 1 nondist.	* this paper
chain	chain	* this paper

Table 1: Summary of polynomial cases: CQ is a complete language for rewritings for the listed subcases of CQ queries and views. The cases with asterisk (\*) assume binary base predicates.

appears in the head. We shall use names beginning with lower-case letters for constants and relations, and names beginning with upper-case letters for variables. We use  $V, V_1, \dots, V_m$  to denote *views* that are defined by conjunctive queries on the base relations. We say that a CQ is *minimized* if there are not redundant subgoals, i.e., if we delete any subgoal then we obtain a query which is not equivalent to the original query. In the rest of this paper, we consider wlog minimized queries and views.

A *relational structure* is a set of atoms over a *domain* of variables and constants. A relational atom with constants in its arguments is called a *ground atom*. A database instance is a relational structure with only ground atoms. The body of a conjunctive query can be also viewed as a relational structure. A *homomorphism* is a mapping from the variables and constants of a relational structure  $S_1$  to the variable and constants of another relational structure  $S_2$  so that an atom of  $S_1$  maps on an atom of  $S_2$  with the same predicate name.

**DEFINITION 2.1.** (*canonical database of query*) A canonical database  $D_Q$  of conjunctive query  $Q$  is derived by freezing the variables of  $Q$  to distinct constants and adding in  $D_Q$  exactly all frozen subgoals in the body of  $Q$ .

For minimized queries canonical database is unique up to renaming. We will use the notation  $D_Q$  to denote the canonical database of query  $Q$  without mentioning it.

**DEFINITION 2.2.** (*query containment and equivalence*) A query  $Q_1$  is contained in a query  $Q_2$ , denoted  $Q_1 \sqsubseteq Q_2$ , if for any database  $D$  of the base relations, the answer computed by  $Q_1$  is a subset of the answer by  $Q_2$ , i.e.,  $Q_1(D) \subseteq Q_2(D)$ . The two queries are equivalent, denoted  $Q_1 \equiv Q_2$ , if  $Q_1 \sqsubseteq Q_2$  and  $Q_2 \sqsubseteq Q_1$ .

Chandra and Merlin [CM77] show that a conjunctive query  $Q_1$  is contained in another conjunctive query  $Q_2$  if and only if there is *containment mapping* from  $Q_2$  to  $Q_1$ . A containment mapping is a *homomorphism* which maps the head and all the subgoals in  $Q_2$  to  $Q_1$ . It maps each variable to either a variable or a constant, and maps each constant to the same constant.

**DEFINITION 2.3.** (*expansion of a query using views*) The expansion of a query  $P$  on a set of views  $\mathcal{V}$ , denoted  $P^{exp}$ , is obtained from  $P$  by replacing all the views in  $P$  with their corresponding base relations. Existentially quantified variables (i.e., nondistinguished variables) in a view are replaced by fresh variables in  $P^{exp}$ .

We denote by  $\mathcal{V}(D)$  the result of computing the views on database  $D$ , i.e.,  $\mathcal{V}(D) = \bigcup_{V \in \mathcal{V}} V(D)$ .

**DEFINITION 2.4.** (*equivalent rewritings*) Given a query  $Q$  and a set of views  $\mathcal{V}$ , a query  $P$  is an equivalent rewriting of query  $Q$  using  $\mathcal{V}$ , if  $P$  uses only the views in  $\mathcal{V}$ , and for any database  $D$  on the schema of the base relations it holds:  $P(\mathcal{V}(D)) = Q(D)$ .

For conjunctive queries and views the following is shown to be an equivalent definition.

**DEFINITION 2.5.** (*equivalent rewritings*) Given a query  $Q$  and a set of views  $\mathcal{V}$ , a query  $P$  is an equivalent rewriting of query  $Q$  using  $\mathcal{V}$ , if  $P$  uses only the views in  $\mathcal{V}$ , and  $P^{exp}$  is equivalent to  $Q$ , i.e.,  $P^{exp} \equiv Q$ .

A CQ query is called *chain query* if it is defined over binary predicates and also the following holds: The body contains as subgoals a number of binary atoms which if viewed as labeled graph (since they are binary) they form a directed path and the start and end nodes of this path are the arguments in the head. For an example, this is a chain query:  $q(X, Y) : -a(X, Z_1, b(Z_1, Z_2), c(Z_2, Y))$ .

**2.2 Determinacy** For two databases  $D_1, D_2$ ,  $\mathcal{V}(D_1) = \mathcal{V}(D_2)$  means that for each  $V_i \in \mathcal{V}$  it holds  $V_i(D_1) = V_i(D_2)$ .

**DEFINITION 2.6.** (*views determine query*) Let query  $Q$  and views  $\mathcal{V}$ . We say that  $\mathcal{V}$  determines  $Q$  if the following is true: For any pair of databases  $D_1$  and  $D_2$ , if  $\mathcal{V}(D_1) = \mathcal{V}(D_2)$  then  $Q(D_1) = Q(D_2)$ .

Thus if a set of views  $\mathcal{V}$  determines a query  $Q$ , then, given a view instance  $I_{\mathcal{V}}$  we know that for any database  $D$  such that  $I_{\mathcal{V}} = \mathcal{V}(D)$  the answer to the query  $Q(D)$  depends only on  $I_{\mathcal{V}}$ , hence we write  $Q(I_{\mathcal{V}})$  to denote it.

The following example shows a view set and two queries, one query being determined by the view set the other query not.

EXAMPLE 2.1. (i) Consider query:

$$Q : q(X, Y) : -a(X, Z_1), a(Z_1, Z_2), b(Z_2, Y).$$

and views:

$$V_3 : v_3(X, Y) : -a(X, Z'_1), a(Z'_1, Z_2), b(Z_2, Y).$$

$$V_4 : v_4(X) : -b(X, X).$$

View set  $\{V_3, V_4\}$  determines  $Q$  because  $V_3$  is equivalent to  $Q$ .

(ii) Now consider query

$$Q' : q'(X, Y) : -a(X, X), b(X, Y).$$

View set  $\{V_3, V_4\}$  does not determine  $Q'$  and also there is no rewriting of  $Q'$  using  $\{V_3, V_4\}$ . To see that  $\{V_3, V_4\}$  does not determine  $Q'$ , consider the databases  $D_1 = \{a(x, z'_1), a(z'_1, z_2), b(z_2, y)\}$  and  $D_2 = \{a(x, x), b(x, y)\}$ . The output of the view computation is the same, i.e.,  $\mathcal{V}(D_1) = \mathcal{V}(D_2) = \{v_3(x, y)\}$  but on  $D_2$  query  $Q'$  computes  $Q'(D_2) = \{q'(x, y)\}$  and on  $D_1$  query  $Q'$  computes the empty set.

DEFINITION 2.7. (complete language for rewritings) Let  $\mathcal{L}_Q$  and  $\mathcal{L}_V$  and  $\mathcal{L}$  be query languages. We say that a language  $\mathcal{L}$  is complete for  $\mathcal{L}_V$ -to- $\mathcal{L}_Q$  rewritings if the following is true for any query  $Q$  in  $\mathcal{L}_Q$  and set of views  $\mathcal{V}$  in  $\mathcal{L}_V$ : Suppose  $\mathcal{V}$  determines  $Q$ ; then there is a query  $R$  in  $\mathcal{L}$  such that  $R$  is an equivalent rewriting of  $Q$  using  $\mathcal{V}$ .

The following proposition states some easy observation about query and views when the views determine the query.

PROPOSITION 2.1. Let query  $Q$  and views  $\mathcal{V}$  be given by minimized conjunctive queries. Suppose  $\mathcal{V}$  determines  $Q$ .

Let  $Q'$  be query resulting from  $Q$  after deleting one or more subgoals. Let  $D_Q$  and  $D_{Q'}$  be the canonical databases of  $Q$  and  $Q'$  respectively. Then the following hold:

- a)  $\mathcal{V}(D_Q) \neq \mathcal{V}(D_{Q'})$ .
- b) For any database  $D$ , the constants in the tuples in  $Q(D)$  is a subset of the constants in the tuples in  $\mathcal{V}(D)$ .
- c) All base predicates appearing in the query definition appear also in the views (but not necessarily vice versa).
- d)  $\mathcal{V}(D_Q) \neq \emptyset$ .

*Proof.* The proof of the first part of the claim is by contradiction: If not, then  $\mathcal{V}(D_Q) = \mathcal{V}(D_{Q'})$  and consequently  $Q(D_Q) = Q(D_{Q'})$ . Hence there is a homomorphism from  $Q$  to  $D_{Q'}$  which yields a containment mapping from the subgoals of  $Q$  to the subgoals of  $Q'$ , which is a contradiction. For the second part, suppose  $x$  is a constant in  $Q(D)$  which does not appear in  $\mathcal{V}(D)$ . Then let us construct  $D_1$  and  $D_2$  to be isomorphic to  $D$  only that in  $D_1$  we have renamed the constant  $x$  to  $c$  where  $c$  is not used again in either  $D_1$  or  $D_2$ . Now we have  $\mathcal{V}(D_1) = \mathcal{V}(D_2)$  but there is a tuple in  $Q(D_1)$  which contains  $c$  and there is no such tuple in  $Q(D_2)$ . Hence  $Q(D_1) \neq Q(D_2)$  contradiction.

To prove the third part, suppose  $p_i$  is the predicate name which appears in the query but does not appear in the views definition. Consider the canonical database  $D_Q$  of the query and a database  $D'$  which results from  $D_Q$  after deleting any fact  $p_i(t)$ . Then on both  $D$  and  $D'$  the views compute the same relations but the query does not. The fourth part is a consequence of a similar construction, now  $D'$  is empty. Then again, on both  $D$  and  $D'$  the views compute the same relations but the query does not.  $\dashv$

It is easy to show that if there is an equivalent rewriting of a query using a set of views then this set of views determine the query.

PROPOSITION 2.2. Let  $Q$  and  $\mathcal{V}$  be query and views which are conjunctive. If there is an equivalent rewriting of  $Q$  using  $\mathcal{V}$  then  $\mathcal{V}$  determines  $Q$ .

*Proof.* Let  $P$  be an equivalent rewriting of  $Q$  using  $\mathcal{V}$ . Let  $D_1$  and  $D_2$  be databases such that  $\mathcal{V}(D_1) = \mathcal{V}(D_2)$ . Then  $P(\mathcal{V}(D_1)) = P(\mathcal{V}(D_2))$ . Since  $P$  is an equivalent rewriting, this yields that  $Q(D_1) = Q(D_2)$ .  $\dashv$

### 3 Complexity of Query Answering

The following theorem shows that, in the case the views determine the query, then the query answering problem is in  $NP \cap coNP$ .

THEOREM 3.1. Let query  $Q$  and views  $\mathcal{V}$  be given by conjunctive queries. Suppose  $\mathcal{V}$  determines  $Q$ . If we are given a view instance  $I_V$  such that there exists a database  $D$  for which  $I_V = \mathcal{V}(D)$ , then computing  $Q(I_V)$  is in  $NP \cap coNP$ .

*Proof.* Let a tuple  $t$  be given. We want to decide whether  $t \in Q(I_V)$  or not. We will show that for every  $I_V$  there exists at least one database  $D_h$  of polynomial size such that  $I_V = \mathcal{V}(D_h)$ . Hence we can decide by computing  $Q(D_h)$ .

We construct a database  $D'$  over the base relations. We “expand”  $I_V$  to  $D'$  (recall that  $I_V$  is over the schema of the views) by replacing each view tuple by

a set of tuples of base relations which are the frozen subgoals of this view's definition. More specifically, nondistinguished variables in the view definition are frozen to fresh constants that are not used for any other frozen variable. (This construction is similar to the construction which obtains the expansion of a rewriting.)

Let  $D$  be any database such that  $I_V = \mathcal{V}(D)$ . Clearly there is a homomorphism from  $D'$  to  $D$ . Also, for any database  $D_i$  such that  $\mathcal{V}(D) = \mathcal{V}(D_i)$  if  $D_i$  is minimal (in the sense that if we delete one tuple from  $D_i$  then  $\mathcal{V}(D) \neq \mathcal{V}(D_i)$ ), then  $D_i$  is a homomorphic image of  $D'$ . Thus, consider all databases that are homomorphic images of  $D'$ . Among those there exists at least one, say  $D_h$ , such that  $I_V = \mathcal{V}(D_h)$ . To compute  $Q(I_V)$  just compute  $Q(D_h)$ .

The above reasoning puts the problem in  $NP \cap coNP$ : The size of any homomorphic image of  $D'$  is polynomial in the size of  $I_V$ . Thus, we guess a homomorphic image  $D_h$  of  $D'$ . Then in polynomial time, we compute the views  $\mathcal{V}$  on  $D_h$  and verify that it is equal to  $I_V$ . Finally we compute  $Q$  on  $D_h$ .  $\dashv$

#### 4 Canonical Rewriting

In this section we show that given a query and views that are defined by conjunctive queries, then there is a particular conjunctive query  $R_c$  which uses only view atoms as subgoals which has the property: If there is an equivalent rewriting then  $R_c$  is an equivalent rewriting.

Let  $D_Q$  be the canonical database of  $Q$ . We compute the views on  $D_Q$  and get view instance  $\mathcal{V}(D_Q)$  [ALU01]. We construct *canonical rewriting*  $R_c$  as follows. The body of  $R_c$  contains as subgoals exactly all unfrozen view tuples in  $\mathcal{V}(D_Q)$  and the tuple in the head of  $R_c$  is as the tuple in the head of query  $Q$ . Here is an example which illustrates this construction.

EXAMPLE 4.1. *Suppose we have the query:*

$$Q : q(X, Y) : -a(X, Z_1), a(Z_1, Z_2), b(Z_2, Y).$$

*and the views  $\mathcal{V}$ :*

$$V_1 : v_1(X, Z_2) : -a(X, Z_1), a(Z_1, Z_2).$$

$$V_2 : v_2(X, Y) : -b(X, Y).$$

*Then  $D_Q$  contains the tuples  $\{a(x, z_1), a(z_1, z_2), b(z_2, y)\}$  and  $\mathcal{V}(D_Q)$  contains the tuples  $\{v_1(x, z_2), v_2(z_2, y)\}$ . And thus,  $R_c$  is the following:*

$$R_c : q(X, Y) : -v_1(X, Z_2), v_2(Z_2, Y).$$

For convenience of reference, we retain in  $R_c$  the names of the variables used in  $Q$ . Observe that all variables of  $R_c$  are also variables of  $Q$  but not necessarily vice versa.

PROPOSITION 4.1. *Let  $Q$  and  $\mathcal{V}$  be conjunctive query and views and  $R_c$  be the canonical rewriting. Then the following hold: a) Query  $Q$  is contained in the expansion  $R_c^{exp}$  of  $R_c$ . b) If there is an equivalent rewriting of  $Q$  using  $\mathcal{V}$  then the canonical rewriting  $R_c$  is such an equivalent rewriting.*

*Proof.* a) By construction of  $R_c$  there is a containment mapping from its expansion  $R_c^{exp}$  to  $Q$ .

b) Suppose there is an equivalent rewriting  $R$  of  $Q$  using  $\mathcal{V}$ . Then the expansion  $R^{exp}$  of  $R$  is equivalent to  $Q$ . Hence there is a containment mapping from  $R^{exp}$  to  $Q$ , and therefore, there is a homomorphism from  $R^{exp}$  to  $D_Q$  (the canonical database of  $Q$ ). Thus if a view atom  $v(t)$  is in the subgoals of  $R$  then there is a homomorphism from the expansion of  $v(t)$  to  $D_Q$ . This establishes that all subgoals in  $R$  must be view atoms that result from view tuples of  $\mathcal{V}(D_Q)$ . But  $R_c$  contains all view tuples in  $\mathcal{V}(D_Q)$ . Thus, any equivalent rewriting  $R$  contains a subset of the subgoals of  $R_c$ , and hence  $R$  contains  $R_c$  and thus  $Q$  contains  $R_c^{exp}$ .  $\dashv$

#### 5 Query Answering in PTIME

In this section we prove that in certain special cases, if views determine the query then there is an equivalent rewriting, hence, computing the answers to the views can be done in polynomial time.

THEOREM 5.1. *Given are a query  $Q$  and a set of views  $\mathcal{V}$  as in one of the cases below:*

1. *All views with no nondistinguished variables.*
2. *Binary base predicates, chain views and query.*
3. *Binary base predicates, query contains only one variable single binary view with only one nondistinguished variable.*

*Suppose  $\mathcal{V}$  determines  $Q$ . Then there is an equivalent rewriting of  $Q$  using  $\mathcal{V}$ .*

*Proof.* For each special case we prove that the canonical rewriting  $R_c$  is an equivalent rewriting.

Case 1. (no nondistinguished variables in views)

Since there are no nondistinguished variables in view definitions  $R_c^{exp}$  contains exactly the variables of  $R_c$ . By construction, there is a one-to-one mapping from the variables of  $R_c$  to the variables of  $Q$  which can be extended to a containment mapping  $\mu$  from  $R_c^{exp}$  to  $Q$ . Moreover, because of Proposition 2.1,  $\mu$  uses as targets all subgoals of  $Q$ . Since the variables of  $R_c^{exp}$  are exactly the variables on  $R_c$ ,  $\mu$  is one-to-one and onto, hence  $\mu^{-1}$  is a containment mapping from  $Q$  to  $R_c^{exp}$ .

Case 2. (Binary base predicates, chain views and query)

*Preliminary observations:* Let  $R_c^{exp}$  be the expansion of the canonical rewriting  $R_c$ . Let  $D_c^{exp}$  be the canonical database of  $R_c^{exp}$  and  $D_Q$  be the canonical database of  $Q$ . Recall that we keep the same names of variables in  $R_c$  as in  $Q$  and do the same for the constants in  $D_c^{exp}$  and  $D_Q$ . As in  $D_c^{exp}$  there are more constants than those in  $D_Q$ , we call the new constants *fresh* (they come from the frozen variables of the fresh variables in  $R_c^{exp}$ ). Let  $X, Y$  be the variables in the head of  $Q$  and  $x, y$  the corresponding constants in  $D_Q$ .  $R_c^{exp}$  is an acyclic (no cycles if the body is viewed as a graph) query (otherwise there is no containment mapping from  $R_c^{exp}$  to  $Q$ ). Let  $\mu$  be a containment mapping from  $R_c^{exp}$  to  $Q$ . We may view  $\mu$  also as a homomorphism from  $D_c^{exp}$  to  $D_Q$ . We consider the expansion  $v_i^{exp}(t)$  of each view subgoal  $v_i(t)$  separately in  $R_c^{exp}$ , we observe that its image on  $Q$  under  $\mu$  is a chain isomorphic to  $v_i^{exp}(t)$ ; hence for every  $v_i^{exp}(t)$ ,  $\mu^{-1}$  is a homomorphism. Let  $G$  be the underlined undirected graph of  $D_c^{exp}$  and  $G_c$  be the underlined undirected graph of the canonical database  $D_c$  of  $R_c$ . Note that connectivity of graphs  $G$  and  $G_c$  are related because all views definitions have a body which is connected if seen as a graph. Thus, each connected component of  $G_c$  creates a connected component of  $G$  and vice versa.

The proof uses connectivity arguments on  $G$ . Let  $x$  and  $y$  be the two constants that are the frozen distinguished variables of  $R_c^{exp}$ . Then  $D_c^{exp}$  can be viewed as a graph with three parts as concerns connectivity of  $G$ : (i) a connected component of  $G$  that includes  $x$ , (ii) a connected component of  $G$  that includes  $y$  and (iii) the remaining part of  $G$ . We have two cases: a) the parts (i) and (ii) are the same, i.e.,  $x$  and  $y$  belong to the same connected component of  $G$  and b) the parts (i) and (ii) are disconnected.

For the first case, we argue that there is a rewriting. We will present in detail the case where the query is also a chain query. It is done by induction on the distance between  $x$  and  $y$  on the graph  $G$ . If the distance is one then there is necessarily a view on the tuple  $(x, y)$  and this view consists the rewriting because  $\mu^{-1}$  provides the containment mapping from  $R_c^{exp}$  to  $Q$ . Inductive hypothesis: If the distance is less than  $n$  then the views on the path from  $x$  to  $y$  consist a rewriting. Suppose the distance is  $n$ . Then, there is a constant  $c$  in  $D_c^{exp}$  in distance less than  $n$  from  $x$  and hence by inductive hypothesis, there is a rewriting of the query  $Q'$  which consists of the first but one subgoals of the query  $Q$  (since it is a chain query, we may view its subgoals as having a natural order). Then the  $n$ -th view added to this rewriting creates a rewriting of  $Q$  due to the isomorphism property.

For the second case we argue that this leads to contradiction. We have the following cases:

a) The connected component in  $G$  which contains

$x$  is empty. This means that in  $G_c$  also the connected component that contains  $X$  is empty and hence there is no tuple in  $\mathcal{V}(D_Q)$  which contains  $X$ . Then we construct the following two database instances:  $D_1$  is a copy of  $D_Q$  and  $D_2$  is a copy of  $D_Q$  with  $x$  and the tuples that contains it being deleted. Clearly  $\mathcal{V}(D_1) = \mathcal{V}(D_2)$  but  $Q(D_1) \neq Q(D_2)$ , contradiction.

b) There are only two connected components in  $G$ , one which contains  $x$  and one which contains  $y$ . Then we construct the following two database instances:  $D_1$  is a copy of  $D_Q$  and  $D_2$  is the disjoint union of two database instances, one for each component of  $G$ . For component  $G_x$  we take the homomorphic image  $\mu(G_x)$  and same for the other component. Note that  $\mu(G_x)$  is isomorphic to the longest path in  $G_x$  because all homomorphisms that produce view tuples are isomorphisms in our case. We claim that  $\mathcal{V}(D_1) = \mathcal{V}(D_2)$ . Suppose not. Then there is a view tuple in  $\mathcal{V}(D_2)$  which is not in  $\mathcal{V}(D_1)$ . We argue on the graph  $G_c$  inductively on the distance from  $x$ . If this tuple contains  $x$  then this means that there is a homomorphism of some view definition on  $D_2$  and not on  $D_1$ . However this homomorphism on  $D_2$  would have created a view tuple in component  $G_x$  and hence would have created a view tuple in  $D_1$  too. This argument can be inductively (on the distance of  $x'$  from  $x$  in  $G_c^x$ , the component of  $G_c$  that corresponds to  $G_x$ .) applied to all  $x'$  where  $x'$  is a constant in a view tuple in component  $G_x$ . Thus what remains is to assume that there is a view tuple in  $D_2$  which does not contain any constants also appearing in view tuples in  $G_c^x$ . Then this means that this view tuple appears in a third connected component in  $G_c$ , contradiction because we assumed there is not any such component. To finish, this case, note that a consequence of the claim is that  $Q(D_1) \neq Q(D_2)$ , contradiction.

c) There are in  $G$  at least three connected components and two of those contain  $x$  and  $y$  each. In this case, we consider  $D_1, D_2$  as in previous case (with the only difference that  $D_2$  is the disjoint union of the homomorphic image under  $\mu$  for each component of  $G$ ) but we are not done yet with the construction of counterexample database instances. We consider tuples that are in  $\mathcal{V}(D_2)$  and not in  $\mathcal{V}(D_1)$ . As we showed above such a tuple does not contain constants in  $G_c^x$ . Let  $v(t)$  be such a tuple. Since  $D_2$  homomorphically maps on  $D_Q$ , a copy  $v(t')$  of  $v(t)$  exist in  $\mu(G_x)$  hence in  $\mathcal{V}(D_Q)$  hence in  $D_2$ . If the component of  $D_2$  in which  $v(t')$  appears is such that no "fresh" tuple from  $G_y$  appears, then we merge this component (its homomorphic image actually) with  $G_x$  and create  $D_2'$ . We do the same for each tuple that satisfies this condition. Thus we need to argue on tuples  $v(t')$  that do not satisfy this condition. We do that by an inductive argument on the length of the expansion of view tuple  $v(t')$  and in each inductive step we create a connected component in the counterexample database

which is shorter than the component we created in the previous inductive step.

Case 3. (Binary base predicates, query contains only one variable single binary view with only one non-distinguished variable)

Let the distinguished variables of the view  $V$  be called  $X, Y$  and the nondistinguished  $Z$ . In this case the canonical rewriting is  $R_c : q'(A) : \neg v(A, A)$ . We consider the expansion  $R_c^{exp}$  of  $R_c$  and the canonical database  $D_c^{exp}$  of  $R_c^{exp}$ . Note that in  $R_c^{exp}$  ( $D_c^{exp}$  respectively) there are only two variables (constants respectively), let us call them  $A$  and  $B$  ( $a$  and  $b$  respectively). Suppose there is no mapping from the query to  $R_c^{exp}$ . Then there is a base relation atom  $r(X)$  in the body of the query and there is no atom  $r(X)$  in the body of  $R_c^{exp}$ . Then there is a database instance  $D$  which is a homomorphic image of the view definition with the property:  $D$  contains two constants  $c, d$  and  $V(D)$  contains tuple  $(c, d)$  and there is no tuple  $r(c, c)$  in  $D$ . To construct this database just take the view definition and identify variable  $Z$  to  $Y$ . Then rename  $X$  to  $c$  and  $Y$  to  $d$ . Symmetrically, there is also a database instance  $D'$  which is a homomorphic image of the view definition with the property:  $D'$  contains two constants  $c', d'$  and  $V(D')$  contains tuple  $(d', c')$  and there is no tuple  $r(c', c')$  in  $D'$ . To construct this database just take the view definition and identify variable  $Z$  to  $X$ . Then rename  $Y$  to  $c'$  and  $X$  to  $d'$ .

We construct two databases  $D_1$  and  $D_2$ .  $D_1$  is constructed by the disjoint union of  $D_c^{exp}$  with  $D$  and  $D'$  and moreover by identifying  $a$  with  $c$  and  $c'$  and then  $b$  with  $d$  and  $d'$ .  $D_2$  is the union of  $D_1$  with the query canonical database and we identify the constant in the query with  $a$ . Now both  $V(D_1)$  and  $V(D_2)$  contain all four possible tuples but  $Q(D_2)$  contains  $(a, a)$  and  $Q(D_1)$  does not, hence contradiction.  $\dashv$

## 6 Determinacy and query equivalence

The problem that we investigate in this paper relates determinacy to query rewriting. Since the problem for the CQ case remains open, we address in this section the question whether a simpler (and probably easier to resolve) variant of the problem may relate determinacy to query equivalence. First we ask: If  $Q_1$  is contained in  $Q_2$  and  $Q_2$  determines  $Q_1$ , then are  $Q_1$  and  $Q_2$  equivalent? The following simple example shows that this statement does not hold: Let  $Q_1 : q_1(X, X) : \neg a(X, X)$  and  $Q_2 : q_2(X, Y) : \neg a(X, Y)$ . Obviously  $Q_1$  is contained in  $Q_2$ . Also  $Q_2$  determines  $Q_1$  because there is an equivalent rewriting of  $Q_1$  using  $Q_2$ , it is  $R : q(X, X) : \neg q_2(X, X)$ . But  $Q_1$  and  $Q_2$  are *not* equivalent.

We add some stronger conditions: Suppose in addition that there is a containment mapping that uses as targets all subgoals of  $Q_1$  and this containment

mapping maps the variables in the head one-to-one. Still the following counterexample shows that we can not conclude that  $Q_1$  and  $Q_2$  are equivalent.

EXAMPLE 6.1. In this example we have two queries:

$Q_1 : q_1(X, Y, Z, W, A, B) : \neg r(Y, X), s(Y, X), r(Z, W), s(Z, Z_1), s(Z_1, Z_1), s(Z_1, W), s(A, A_1), s(A_1, A_1), s(A_1, B)$ .

and

$Q_2 : q_2(X, Y, Z, W, A, B) : \neg r(Y, X), s(Y, X), r(Z, W), s(Z, Z_1), s(Z_1, Z_2), s(Z_2, W), s(A, A_1), s(A_1, A_1), s(A_1, B)$ .

Clearly  $Q_1$  is contained in  $Q_2$ . Also  $Q_2$  determines  $Q_1$  because there is an equivalent rewriting of  $Q_1$  using  $Q_2$ :

$R : q'_1(X, Y, Z, W, A, B) : \neg q_2(X, Y, Z, W, A, B), q_2(X_1, Y_1, Z_1, W_1, Z, W)$ .

Moreover there is a homomorphism from  $Q_2$  to  $Q_1$  that uses all subgoals of  $Q_1$  and is one-to-one on the head variables. But  $Q_1$  and  $Q_2$  are not equivalent.

Finally, in order to be convinced that  $R$  is a rewriting, let us consider the expansion

$R^{exp} : q'_1(X, Y, Z, W, A, B) : \neg r(Y, X), s(Y, X), r(Z, W), s(Z, Z_1), s(Z_1, Z_2), s(Z_2, W), s(A, A_1), s(A_1, A_1), s(A_1, B), r(Y_1, X_1), s(Y_1, X_1), r(Z_1, W_1), s(Z_1, Z'_1), s(Z'_1, Z'_2), s(Z'_2, W_1), s(Z, A'_1), s(A'_1, A'_1), s(A'_1, W)$ .

Then homomorphism  $\mu_1$  is a containment mapping from  $R^{exp}$  to  $Q_1$  and homomorphism  $\mu_2$  is a containment mapping from  $Q_1$  to  $R^{exp}$ :

$\mu_1 : \{X \rightarrow X, Y \rightarrow Y, Z \rightarrow Z, W \rightarrow W, A \rightarrow A, B \rightarrow B, Z'_1 \rightarrow Z_1, Z'_2 \rightarrow Z_1, A'_1 \rightarrow A_1, A'_2 \rightarrow Z_1, X_1 \rightarrow X, Y_1 \rightarrow Y, Z_1 \rightarrow Z, W_1 \rightarrow W, Z'_1 \rightarrow Z_1, Z'_2 \rightarrow Z_1\}$

$\mu_2 : \{X \rightarrow X, Y \rightarrow Y, Z \rightarrow Z, W \rightarrow W, A \rightarrow A, B \rightarrow B, A_1 \rightarrow A'_1, Z_1 \rightarrow A'_1\}$

Finally we add another condition which we denote by  $Q_2(D_1) \subseteq_s Q_2(D_2)$ , where  $D_1, D_2$  is the canonical databases of  $Q_1, Q_2$  respectively.

We need first explain the notation  $Q(D_1) \subseteq_s Q(D_2)$  which in general expresses some structural property of databases  $D_1$  and  $D_2$  with respect to  $Q$  and this property is invariant under renaming. We say that  $Q(D_1) \subseteq_s Q(D_2)$  holds if there is a renaming of the constants in  $D_1, D_2$  such that  $Q(D_1) \subseteq Q(D_2)$ . For an example, say we have query  $Q : q(X, Y) : \neg r(X, Y)$  and three database instances  $D_1 = \{r(1, 2), r(2, 3)\}$ ,  $D_2 = \{r(a, b), r(b, c)\}$  and  $D_3 = \{r(a, b), r(a, c)\}$ . Then it holds that  $Q(D_1) \subseteq_s Q_1(D_2)$  and  $Q(D_1) \subseteq_s Q(D_2)$  because there is a renaming of  $D_2$  (actually here  $D_1, D_2$  are isomorphic) such that  $Q(D_1) \subseteq Q_1(D_2)$  and  $Q(D_1) \subseteq Q(D_2)$ . But the following does not hold:  $Q(D_3) \subseteq_s Q(D_2)$ .

We may also allow some constants in  $D_1, D_2$  that are special as concerns renaming. Although we need incorporate these constants in the notation, we will keep (slightly abusively) the same notation here since we always mean the same constants. Thus let us return to queries  $Q_1, Q_2$  and their canonical databases  $D_1, D_2$  respectively. Then by  $Q_2(D_1) \subseteq_s Q_2(D_2)$  we mean in

addition that (i) the frozen variables in the head of the queries are identical component-wise, i.e., if in the head of  $Q_1$  we have tuple  $(X_1, \dots, X_m)$  then in the head of  $Q_2$  we also have same tuple  $(X_1, \dots, X_m)$  and in both  $D_1, D_2$  these variables freeze to constants  $x_1, \dots, x_m$  and (ii) if we need to rename, then we are not allowed to rename the constants  $x_1, \dots, x_m$ .

We introduce a new problem which relates determinacy to query equivalence:

*Determinacy and query equivalence:* Let  $Q_1, Q_2$  be conjunctive queries. Suppose  $Q_2$  determines  $Q_1$ , and  $Q_1$  is contained in  $Q_2$ . Suppose also that the following hold: a) there is a containment mapping from  $Q_2$  to  $Q_1$  which (i) uses as targets all subgoals of  $Q_1$  and (ii) maps the variables in the head one-to-one, and b)  $Q_2(D_1) \subseteq_s Q_2(D_2)$ , where  $D_1, D_2$  are the canonical databases of  $Q_1, Q_2$  respectively. Then are  $Q_1$  and  $Q_2$  equivalent?

Theorem 6.1 states that if CQ is complete for CQ-to-CQ rewritings then the answer to the above question is "yes".

**THEOREM 6.1.** *For the following two statement it holds: Statement (A) implies statement (B).*

A) Let  $Q$  and  $\mathcal{V}$  be conjunctive query and views. Suppose  $\mathcal{V}$  determines  $Q$ . Then there is an equivalent rewriting of  $Q$  using  $\mathcal{V}$ .

B) Let  $Q_1, Q_2$  be conjunctive queries. Suppose  $Q_2$  determines  $Q_1$ , and  $Q_1$  is contained in  $Q_2$ . Suppose also that the following hold: a) there is a containment mapping from  $Q_2$  to  $Q_1$  which (i) uses as targets all subgoals of  $Q_1$  and (ii) maps the variables in the head one-to-one, and b)  $Q_2(D_1) \subseteq_s Q_2(D_2)$ , where  $D_1, D_2$  are the canonical databases of  $Q_1, Q_2$  respectively. Then  $Q_1$  and  $Q_2$  are equivalent.

*Proof.* Suppose statement (A) is true. Since  $Q_2$  determines  $Q_1$  and statement (A) is true, there is an equivalent rewriting of  $Q_1$  using  $Q_2$ . Then the canonical rewriting  $R_c$  is such a rewriting (see Proposition 4.1). Hence  $R_c^{exp}$  is equivalent to  $Q_1$ . Thus, if we prove that  $R_c^{exp}$  is equivalent to  $Q_2$ , then this implies that  $Q_1$  and  $Q_2$  are equivalent. We know that there is a containment mapping from  $Q_2$  to  $R_c^{exp}$ . In order to prove that there is a containment mapping from  $R_c^{exp}$  to  $Q_2$ , we observe that  $Q_2(D_1) \subseteq_s Q_2(D_2)$  implies such a mapping. The reason is that there is a containment mapping from  $R_c^{exp}$  to  $Q_1$  and this mapping produces some tuples in  $Q_2(D_1)$ . Therefore analogous tuples must be produced in  $Q_2(D_2)$ . Hence there is a containment mapping from  $R_c^{exp}$  to  $Q_2$ .  $\dashv$

The *determinacy and query equivalence* question remains open. The following theorem settles a special case where we have replaced condition (b) with a stronger one.

**THEOREM 6.2.** *Let  $Q_1, Q_2$  be conjunctive queries. Suppose  $Q_2$  determines  $Q_1$ , and  $Q_1$  is contained in  $Q_2$ . Suppose also that the following hold: a) there is a containment mapping that uses as targets all subgoals of  $Q_1$  and this containment mapping maps the variables in the head one-to-one, and b)  $Q_2(D_1)$  contains exactly one tuple, where  $D_1$  is the canonical databases of  $Q_1$ . Then  $Q_1$  and  $Q_2$  are equivalent.*

*Proof.* Intuitively the first condition in the statement of the theorem says that  $Q_1$  is a homomorphic image of  $Q_2$ , i.e., formed from  $Q_2$  by identifying variables. Moreover it also says that the distinguished variables are *not* to be identified, so  $Q_1$  is  $Q_2$  with (perhaps) nondistinguished variables identified. Suppose towards contradiction that  $Q_2$  and  $Q_1$  are not equivalent. Then there is a query  $Q'_1$  such that  $Q_1 \sqsubseteq Q'_1 \sqsubseteq Q_2$  and  $Q'_1$  differs from  $Q_1$  only in that  $Q_1$  results from  $Q'_1$  by identifying only two variables (not both distinguished). The following lemma states formally this observation.

**LEMMA 6.1.** *Suppose  $Q_1$  is contained in  $Q_2$  but they are not equivalent. Then there is a query  $Q'_1$  with the properties: a)  $Q'_1$  is contained in  $Q_2$  b)  $Q_1$  is properly contained in  $Q'_1$  and c) the containment mapping  $h$  from  $Q'_1$  to  $Q_1$  is identity for all variables of  $Q'_1$  except that  $h(X_1) = h(X_2) = X_1$ .*

*Proof.* Observe that we conveniently keep the names of the variables in  $Q_1$  and  $Q'_1$  (except  $X_2$  which appears only in  $Q'_1$ ).

Let  $h_1$  be the homomorphism from the subgoals of  $Q_2$  to the subgoals of  $Q_1$ . Based on  $h_1$  we construct homomorphism  $h$  which defines a homomorphic image of  $Q_2$  and has the properties as in the statement of the lemma. Since  $Q_1$  and  $Q_2$  are not equivalent, there are variables  $X_1, X_2$  of  $Q_2$  such that  $h_1(X_1) = h_1(X_2)$ . Then  $h$  is the following: It is the same as  $h_1$  except that for  $h$  we have  $h(X_1) \neq h(X_2)$ . Hence in  $Q'_1$  we have all variable names as in  $Q_1$  and an additional variable  $X_2$ . It is easy to see that  $Q'_1$  properly contains  $Q_1$  and is contained in  $Q_2$ .  $\dashv$

Let  $Q'_1$  be a query with the properties of the lemma above. Consider the canonical database  $D'_1$  of  $Q'_1$  and compute  $Q_2(D'_1)$ . We claim that  $Q_2(D'_1)$  contains at most two tuples. Because:  $Q_2(D_1)$  (which contains only one tuple) is a homomorphic image of  $Q_2(D'_1)$  and thus (because  $Q'_1$  has the property in Lemma 6.1 wrto  $Q_1$ , i.e, the homomorphism  $h$  which maps  $Q'_1$  to  $Q_1$  is the identity except what concerns the variables  $X_1$  and  $X_2$ ) there may be only one additional tuple in  $Q_2(D'_1)$ , the one that contains  $X_2$ .

Thus we have two cases: a)  $Q_2(D'_1)$  contains one tuple. b)  $Q_2(D'_1)$  contains two tuples. In the first case,  $Q_2(D'_1) = Q_2(D_1)$ , hence  $Q_1(D'_1) = Q_1(D_1)$  which is



false, hence a contradiction. In the second case, we construct database  $D_1'' : D_1''$  is  $D_1$  with additional facts the frozen subgoals that contain  $X_2$  in  $Q_1'$  – observe that  $D_1'$  is a subset of  $D_1''$ . We observe that  $Q_2(D_1'')$  contains at most two tuples for the same reason for which  $Q_2(D_1')$  contains at most two tuples. Hence  $Q_2(D_1'') = Q_2(D_1')$  and consequently  $Q_1(D_1'') = Q_1(D_1')$  a contradiction.  $\dashv$

Theorem 6.2 covers interesting special cases that include: a) queries do not contain self-joins and b) query  $Q_1$  contains a single variable.

## 7 Principal view set reduction

In this section we first define the concept of principal view set for query  $Q$ . Informally a view set  $\mathcal{V}$  is principal for  $Q$  if in  $\mathcal{V}(D_Q)$  there exists one view  $V_p$  which we call *principal view for  $Q$*  for which it holds: if there is an equivalent rewriting of  $Q$  using  $\mathcal{V}$  then a projection of the principal view is such a rewriting. In Theorem 7.1 we reduce the general problem to the special case where the view set is principal.

Let  $D_p^{exp}$  be the canonical database of the expansion  $V_p^{exp}$  of the principal view  $V_p$ . We use the notation  $V_p(D_Q) \subseteq_s V_p(D_p^{exp})$  where again we assume that a set of frozen variables are shared between  $D_Q$  and  $D_p^{exp}$  and are not allowed to be renamed. This is the set of variables in the head of  $V_p$  which includes the set of variables in the head of  $Q$  and is a subset of the set of variables in  $Q$ .

**DEFINITION 7.1.** (*principal view set*) Let  $Q$  and  $\mathcal{V}$  be  $CQ$  query and views.

- A view  $V_p \in \mathcal{V}$  is called *principal view for  $Q$*  if the following hold: a) There is a homomorphism from the definition of  $V_p$  to  $Q$  which uses all subgoals of  $Q$  as targets and is one-to-one for the distinguished variables of  $V_p$ . b) Let  $t$  be the tuple of frozen distinguished variables in  $D_p^{exp}$ . Then all tuples in  $V_p(D_Q)$  use only constants from  $t$  and c)  $V_p(D_Q) \subseteq_s V_p(D_p^{exp})$ .
- A view set is called *principal view set for  $Q$*  if it contains a principal view  $V_p$ .

**EXAMPLE 7.1.** We use the queries  $Q_1, Q_2$  defined in Example 6.1. Thus let  $Q = Q_1$  and let the view set contain a single view  $V = Q_2$ . Then conditions (a) and (b) in Definition 7.1 are true but condition (c) is not.

In order to show that  $V(D_Q) \subseteq_s V(D^{exp})$  does not hold let us consider canonical databases,  $D_Q$ , of query  $Q$  and  $D^{exp}$ , of the expansion of view  $V$ . It is important however to name constants in  $D^{exp}$  according to the targets in  $Q$  of the homomorphism claimed in part (a) in Definition 7.1. Thus these databases are:

$$D_Q = \{r(y, x), s(y, x), r(z, w), s(z, z_1), s(z_1, z_1), s(z_1, w),$$

$$s(a, a_1), s(a_1, a_1), s(a_1, b)\} \\ D^{exp} = \{r(y, x), s(y, x), r(z, w), s(z, z'_1), s(z'_1, z'_2), s(z'_2, w), \\ s(a, a_1), s(a_1, a_1), s(a_1, b)\} \\ \text{We compute } V(D_Q) = \{(x, y, z, w, a, b), (x, y, z, w, z, w)\} \\ \text{and } V(D^{exp}) = \{(x, y, z, w, a, b)\}. \text{ Hence } V(D_Q) \subseteq_s \\ V(D^{exp}) \text{ does not hold, therefore } V \text{ is not a principal view} \\ \text{for } Q \text{ and view set } \{V\} \text{ is not a principal view set for } Q.$$

Given arbitrary query  $Q$  and views  $\mathcal{V}$  we can construct a principal view from the canonical rewriting  $R_c$ . We call this view *canonical view* and denote by  $V_c$ . We construct view  $V_c$  to have as subgoals exactly all subgoals of the expansion of  $R_c$  and in the head of  $V_c$  we have exactly all variables of  $R_c$ .

**EXAMPLE 7.2.** From Example 4.1 we have that the canonical rewriting for  $\{V_1, V_2\}$  and  $Q$  is

$$R_c : q(X, Y) : -v_1(X, Z_2), v_2(Z_2, Y).$$

and hence the canonical view is

$$V_c : v_c(X, Y, Z_2) : -a(X, Z_1), a(Z_1, Z_2), b(Z_2, Y).$$

**PROPOSITION 7.1.** Let  $Q$  and  $\mathcal{V}$  be query and views defined by conjunctive queries. Then the canonical view  $V_c$  is a principal view for  $Q$ .

*Proof.* Clearly  $V_c(D_Q)$  contains at least one tuple  $t$  by construction of  $V_c$ . Moreover  $t$  is obtained by a homomorphism which maps one-to-one the distinguished variables of  $V_c$  and uses all subgoals of  $Q$  according to Proposition 2.1. This proves part (a) of Definition 7.1.

Now observe that any homomorphism from the definition of  $V_c$  to  $D_Q$  creates view tuples in  $\mathcal{V}(D_Q)$ . Suppose there is an homomorphism from the definition of  $V_c$  to  $D_Q$  which does not use constants in  $t$ . Then this homomorphism creates view tuples not in  $R_c$  which is a contradiction because  $R_c$  contains as subgoals all view tuples in  $\mathcal{V}(D_Q)$ . This proves part (b) of the definition. For the same reason, the following leads to contradiction and hence proves part (c) of the definition: If there is a tuple in  $V_p(D_Q)$  and not in  $V_p(D_p^{exp})$  then this yields that a view tuple in  $\mathcal{V}(D_Q)$  does not appear in the subgoals of  $R_c$ .  $\dashv$

The following theorem reduces the general problem to the problem where we have a principal view set.

**THEOREM 7.1.** Let  $Q$  and  $\mathcal{V}$  be query and views. Then there is a principal view set  $\mathcal{V}'$  such that the following hold:

1. If  $\mathcal{V}$  determines  $Q$  then  $\mathcal{V}'$  determines  $Q$ .
2. If there is an equivalent rewriting of  $Q$  using  $\mathcal{V}'$  then there is an equivalent rewriting of  $Q$  using  $\mathcal{V}$ .

*Proof.* View set  $\mathcal{V}'$  is the union of  $\mathcal{V}$  and the set that contains only the canonical view. We proved in Proposition 7.1 that the canonical view is a principal view.

The following proposition is an immediate consequence of Proposition 4.1.

**PROPOSITION 7.2.** *Let  $Q$  and  $\mathcal{V}$  be query and views such that  $\mathcal{V}$  is a principal view set for  $Q$ . If there is an equivalent rewriting of  $Q$  using  $\mathcal{V}$ , then there is an equivalent rewriting which is a projection of the principal view of  $\mathcal{V}$ .*

Proposition 7.2 states that in the case of principal view set, if there is an equivalent rewriting then a projection of the principal view is an equivalent rewriting. Note that in Section 6 we essentially discussed a case where the principal view does not need a projection to produce a rewriting.

## 8 Other Reductions

In this section we prove the following theorem and provide some formal considerations about connectivity too. By  $CQ_{bin}$  we mean the language of CQs where the base predicates are binary. By  $CQ_{Bool,bin}$  we mean the language of  $CQ_{bin}$  where the query is boolean.

**THEOREM 8.1.** *If the language of CQ is complete for  $CQ_{bin}$ -to- $CQ_{Bool,bin}$  rewritings then it is complete for  $CQ$ -to- $CQ$  rewritings.*

### 8.1 Binary base predicates

Here we prove:  
**THEOREM 8.2.** *If the language of CQ is complete for  $CQ_{bin}$ -to- $CQ_{bin}$  rewritings then it is complete for  $CQ$ -to- $CQ$  rewritings.*

The above theorem is an immediate consequence of the following theorem.

**THEOREM 8.3.** *Let  $Q$  and  $\mathcal{V}$  be query and views. Then there is a view set  $\mathcal{V}'$  and a query  $Q'$  defined on binary base predicates such that the following hold:*

1. *If  $\mathcal{V}$  determines  $Q$  then  $\mathcal{V}'$  determines  $Q'$ .*
2. *If there is an equivalent rewriting of  $Q'$  using  $\mathcal{V}'$  then there is an equivalent rewriting of  $Q$  using  $\mathcal{V}$ .*

*Proof.* We construct query  $Q'$  and views  $\mathcal{V}'$  as follows: We introduce for each base predicate  $p_i$  of arity  $r_i$  a collection of  $r_i(r_i - 1)/2$  distinct binary base predicates  $a_{j,j'}^i, j < j' = 1, \dots, r_i$ . From query  $Q$  (view  $V_j$  respectively) we construct query  $Q'$  (view  $V_j'$  respectively) as follows. We replace each occurrence of predicate atom  $p_i(X_1, \dots, X_{r_i})$  by  $r_i(r_i - 1)/2$  binary predicates atoms which in particular are:  $a_{j,j'}^i(X_j, X_{j'}), j < j' = 1, \dots, r_i$ . Intuitively each base predicate is replaced by a clique but the edges of the clique have different predicate names in order to retain the information about the positions of the variables when they are used as arguments in the non-binary predicates.

Proof of part 1. Let  $D_1', D_2'$  be databases over the binary predicates such as  $\mathcal{V}'(D_1') = \mathcal{V}'(D_2')$ . We

construct from  $D_1', D_2'$  database instances  $D_1, D_2$  over the predicates used in  $\mathcal{V}$ . For each homomorphic image of a clique in  $D_i'$  we add in  $D_i$  a tuple accordingly in a way that corresponds to the construction above. The homomorphic mappings of the views  $\mathcal{V}'$  on  $D_i'$  carry over to homomorphic mappings of the views  $\mathcal{V}$  on  $D_i$  and vice versa. Hence  $\mathcal{V}'(D_1') = \mathcal{V}(D_1)$  and  $Q'(D_1') = Q(D_1)$ . Therefore  $\mathcal{V}(D_1) = \mathcal{V}(D_2)$ . This yields that  $Q(D_1) = Q(D_2)$ .

Proof of part 2. If there is an equivalent rewriting of  $Q'$  using  $\mathcal{V}'$  then we claim that the same with the primed view atoms replaced by unprimed view atoms is an equivalent rewriting of  $Q$  using  $\mathcal{V}$ . The containment mappings that are used in the former case carry over to the latter because the distinct binary base predicates introduced take care of the positions of the variables in the non-binary base predicates of the latter case.  $\dashv$

### 8.2 Boolean Queries

Here we prove Theorem 8.1 which is an immediate consequence of the following:

**THEOREM 8.4.** *Let  $Q$  and  $\mathcal{V}$  be query and views over binary predicates. Then there is a view set  $\mathcal{V}'$  and a boolean query  $Q'$  over binary predicates such that the following hold:*

1. *If  $\mathcal{V}$  determines  $Q$  then  $\mathcal{V}'$  determines  $Q'$ .*
2. *If there is an equivalent rewriting of  $Q'$  using  $\mathcal{V}'$  then there is an equivalent rewriting of  $Q$  using  $\mathcal{V}$ .*

*Proof.* Given query  $Q$  of arity  $r$  and views  $\mathcal{V}$  we construct query  $Q'$  and views  $\mathcal{V}'$  as follows. First introduce  $r$  new predicates for new base relations let them be  $a_1, \dots, a_r$ . Query  $Q'$  is a Boolean query with all subgoals in  $Q$  and additional subgoals one for each argument in the head. More specifically, if variable  $X$  is in the head in position  $i$  then we add  $a_i(X, Z)$  where  $Z$  is also a new variable used (different from all other variables used also in  $Q$ ). The views  $\mathcal{V}'$  include all views in  $\mathcal{V}$  and some new views. We add new views one for each  $a_i$  defined by  $v_i(t) : \neg a_i(t)$ .

Now consider two databases on which views  $\mathcal{V}'$  compute the same relations. Since  $\mathcal{V}$  is a subset of  $\mathcal{V}'$  then views  $\mathcal{V}$  compute the same relations on both databases. Hence query  $Q$  computes the same relation on both databases. If the query output is empty on both then query  $Q'$  is false. Otherwise it might be either false or true depending on how the new relations  $v_i, i = 1, 2, \dots$  compute on the two databases. However, since they produce the same output and  $Q$  produces the same output on both, then  $Q'$  is either false on both or true on both. The reason is that  $Q'$  can be rewritten using  $Q$  and the relations  $v_i, i = 1, 2, \dots$ .  $\dashv$

### 8.3 Connectivity

The following example shows the intuition for the result in this section.

EXAMPLE 8.1. Suppose we have query:

$$Q : Q(X) : -r(Y, X), s(Y, X), s(Z, Z_1), s(Z_1, Z).$$

and views  $\mathcal{V}$ :

$$V_1 : v_1(X, Y) : -r(Y, X).$$

$$V_2 : v_2(X, Y) : -s(Y, X), s(Z, Z_1), s(Z_1, Z).$$

Clearly the rewriting  $R_1 : Q(X) : -v_1(X, Y), v_2(X, Y)$  is an equivalent rewriting of  $Q$  using  $\mathcal{V}$ . Note however, that the two last subgoals in the query are (what we call in the definition below semi-covered) such that either they are covered by a single view among the views in the particular view set or not at all. The reason is that these two subgoals do not contain any variables in the view tuples of  $\mathcal{V}(D_Q)$  ( $D_Q$  is the canonical database of  $Q$ ) hence it is not possible to "glue together" several views to "cover" these two subgoals. In that case, we might be able to simplify the problem by reducing it to the following query  $Q'$  and views  $\mathcal{V}$ :

$$Q' : Q'(X) : -r(Y, X), s(Y, X).$$

and views  $\mathcal{V}$ :

$$V'_1 : v'_1(X, Y) : -r(Y, X).$$

$$V'_2 : v'_2(X, Y) : -s(Y, X).$$

Then the same rewriting provides an equivalent rewriting of  $Q'$  using  $\mathcal{V}'$ . We make this formal in this subsection.

DEFINITION 8.1. (Connectivity graph of query) Let  $Q$  be a conjunctive query. The nodes of the connectivity graph of  $Q$  are all the subgoals of  $Q$  and there is an (undirected) edge between two nodes if they share a variable or a constant.

A *connected component* of a graph is a maximal subset of its nodes such that for every pair of nodes in the subset there is a path in the graph that connects them. A *connected component of a query* is a subset of subgoals which define a connected component in the connectivity graph. A graph with only one connected component is called *connected*. A query is *connected* if its connectivity graph is connected.

DEFINITION 8.2. (semi-covered component) Let  $Q$  and  $\mathcal{V}$  be CQ query and views. Let  $G$  be a connected component of query  $Q$ . Suppose that any variable in  $G$  is such that there is no tuple in  $\mathcal{V}(D_Q)$  ( $D_Q$  is the canonical database of  $Q$ ) that contains it. Then we say that  $G$  is a semi-covered component of  $Q$  wrto  $\mathcal{V}$ .

We prove the following which is the main result of this subsection.

THEOREM 8.5. If the language of CQ is complete for CQ-to-CQ rewritings where the query does not contain any semi-covered components wrto the views then it is complete for CQ-to-CQ rewritings.

THEOREM 8.6. Let  $Q$  and  $\mathcal{V}$  be query and views. Then there is a view set  $\mathcal{V}'$  and a query  $Q'$  such that  $Q'$  has no semi-covered components wrto  $\mathcal{V}'$  and the following hold:

1. If  $\mathcal{V}$  determines  $Q$  then  $\mathcal{V}'$  determines  $Q'$ .
2. If there is an equivalent rewriting of  $Q'$  using  $\mathcal{V}'$  then there is an equivalent rewriting of  $Q$  using  $\mathcal{V}$ .

*Proof.* We define:

DEFINITION 8.3. (covering subgoals) Let view  $V$ , query  $Q$  and let  $S$  be a subset of the subgoals of  $Q$ . We say that  $V$  covers  $S$  if there is a homomorphism  $\mu$  from the view definition to  $Q$  such that  $S$  is a subset of  $\mu(v^{exp})$  i.e., a subset of the targets of the view's subgoals under  $\mu$ .

We use the following lemma:

LEMMA 8.1. Let  $Q$  and  $\mathcal{V}$  be conjunctive query and views. Suppose  $\mathcal{V}$  determines  $Q$ . Let  $G_Q$  be a disconnected component of  $Q$  which is semi-covered by  $\mathcal{V}$ . Then there is a view in  $\mathcal{V}$  which contains a connected component which is isomorphic to  $G_Q$ .

*Proof.* Because of Proposition 2.1, all subgoals of  $G_Q$  are targets of some view tuple mapping when we compute  $\mathcal{V}(D_Q)$ .

First we need to prove that in any mapping  $\mu$  from the views to  $D_Q$  there is only one view which covers  $G_Q$ . Towards contradiction, suppose there is a mapping  $\mu$  for which  $G_Q$  is covered by more than one view, say it is covered by views (wlog)  $v_1, v_2$ . This means that the union of  $\mu(v_1)$  and  $\mu(v_2)$  contains all subgoals in  $G_Q$ . Now, we do some construction: First rename all variables in  $\mu(v_1)$  and  $\mu(v_2)$  so that they take names from disjoint sets for each  $\mu(v_i)$ . Then consider canonical databases for  $\mu(v_i)$ 's and  $G_Q$  – for simplicity we keep the same name. We construct a database  $D_1$  by taking  $D_Q$  and deleting  $G_Q$  and adding all  $\mu(v_i)$ 's. Clearly  $D_1$  and  $D_Q$  are such that  $\mathcal{V}(D_1) = \mathcal{V}(D_Q)$  (because component  $G_Q$  did not produce any view tuples, and its replacement  $G'_Q$  is such that  $G_Q$  is a homomorphic image of  $G'_Q$ , hence  $G'_Q$  does not produce any view tuples either). Then  $D_1$  and  $D_Q$  are such that  $\mathcal{V}(D_Q) = \mathcal{V}(D_1)$  but  $Q(D_Q) \neq Q(D_1)$ .  $\dashv$

We construct  $Q'$  and  $\mathcal{V}'$ . Let  $Q'$  be the query which has the same head as  $Q$  and all subgoals of  $Q$  except those that are semi-covered. Let  $\mathcal{V}'$  be a set of views same as the set  $\mathcal{V}$  but in their definition we have deleted the subgoals that cover semi-covered components of the

query. It is easy to see from the above lemma that if there is an equivalent rewriting  $R$  of  $Q'$  using  $\mathcal{V}'$  then there is an equivalent rewriting  $R_1$  of  $Q$  using  $\mathcal{V}$ .

Now the first part in the statement of Theorem 8.6 remains to be proven. Let  $D_1$  and  $D_2$  be databases on which  $\mathcal{V}'$  computes the same answer, i.e.,  $\mathcal{V}'(D_1) = \mathcal{V}'(D_2)$ . We want to construct database instances  $D'_1, D'_2$  such that  $\mathcal{V}(D'_1) = \mathcal{V}(D'_2)$  and  $Q'(D_i) = Q(D'_i, i = 1, 2)$ . It is worth noting that a construction that is straightforward does not work: Construct  $D'_i$  as the disjoint union of  $D_i$  and database  $D_M$  which is a copy of all semi-covered components of  $Q$  (with variables frozen appropriately similar to the case of constructing a canonical database of a query). The reason is that a tuple  $v_i(t)$  might exist which is in  $\mathcal{V}(D'_1)$  and not in  $\mathcal{V}(D'_2)$ . This is possible for tuples  $t$  which contain constants from  $D_M$  – remember that databases  $D_1, D_2$  may be quite different from each other.

*Construction of databases  $D'_1$  and  $D'_2$ .* We will construct databases  $D'_1$  and  $D'_2$  as follows: They are both isomorphic copies of a database  $D$  where  $D$  is the disjoint union of  $D_1$  and  $D_2$  and database  $D_M$ . Observe that we can partition the variables of  $D$  into three subsets  $A_1, A_2, A_3$  which are those that belong to  $D_1$ , those that belong to  $D_2$  and those that belong to  $D_M$ . By hypothesis, we have that  $\mathcal{V}'$  computes the same view tuples on  $D_1$  and  $D_2$ . Thus, we can rename the constants in  $D$  so that we create two databases  $D'_1$  and  $D'_2$ : In  $D'_1$  we do not do any renaming. In  $D'_2$ , we swap names among constants that are contained in view tuples when we are computing views  $\mathcal{V}'(D_1)$  and  $\mathcal{V}'(D_2)$ . More precisely, if view tuple  $v'_j(x_1, x_2, x_3)$  is in  $\mathcal{V}'(D_1)$  then a "corresponding" view tuple  $v'_j(y_1, y_2, y_3)$  is in  $\mathcal{V}'(D_2)$ . We swap  $x_i$  for  $y_i$ . This kind of swap would run into inconsistencies if it were not true that  $\mathcal{V}'(D_1) = \mathcal{V}'(D_2)$ . Hence it is feasible in our case.

Now it is easy to show that on  $D'_1$  and  $D'_2$ , views  $\mathcal{V}$  compute the same tuples (notice that although  $D'_1$  and  $D'_2$  are isomorphic, because of the renaming, they might not compute the same set of view tuples). Hence the answers of  $Q$  are the same on  $D'_1$  and  $D'_2$ . This implies in a straightforward way that the answers of  $Q'$  are the same on  $D_1$  and  $D_2$ .

## 9 Conclusion

We have investigated the problem whether CQ is complete for CQ-to-CQ rewritings. We have presented a number of special cases for which CQ is complete. On the other end, we have reduced the general problem to more restricted variants. A new problem is introduced in Section 6 that relates determinacy to query equivalence and seems "easier" to resolve than the original problem. However this also remains open, since here we have solved only a (pretty broad) special case of it. Reducing the original problem to the *determi-*

*nacy and query equivalence* problem is another open question. Further we provided a reduction in Section 7 which shows that the problem can be reduced to the case where a projection of a special view provides an equivalent rewriting if there exists one.

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