# Nonmonotonic Inheritance, Argumentation and Logic Programming

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**Abstract.** We study the conceptual relationship between the semantics of nonmonotonic inheritance reasoning and argumentation. We show that the credulous semantics of nonmonotonic inheritance can be captured by the stable semantics of argumentation. Further, we argue that grounded semantics of argumentation provides the baseline of scepticism by showing that it constitutes a part of both ideally skeptical semantics and iterative skeptical semantics of inheritance network. In addition, we present a modular transformation of nonmonotonic inheritance networks into equivalent extended logic programs.

## 1 Introduction

Argument-based approaches to nonmonotonic reasoning have been intensively studied and became prominent in AI and Logic Programming [6,20,23,2,15] just recently. But reasoning based on arguments represented as paths, has been studied in nonmonotonic inheritance reasoning, a specific field of nonmonotonic reasoning, from the very first day [30] and then in [13,14,26,27,28,25,24,11]. Path-based approaches to nonmonotonic inheritance networks are widely accepted because they are intuitive and easy to implement.

The interesting and surprising problem here is that argument-based semantics of nonmonotonic inheritance network [13,14,26,27,28,25,24,11] and general argumentation reasoning [6,20,23,2,15] seem to have conceptually little in common. Touretzky et al. went so far to claim that one of the fundamental principle of argumentation - *the use of reinstater* - can not be applied in nonmonotonic inheritance reasoning [28].

The relation between nonmonotonic inheritance reasoning and more general frameworks to nonmonotonic reasoning like default logic, autoepistemic logic, logic programming has been intensively studied in [3,4,12,9,10,22,16]. The basic idea of these works is to find a way to translate a nonmonotonic inheritance network into a "equivalent" theory of the respected nonmonotonic logic. But all of these transformations do not preserve the original semantics of nonmonotonic inheritance networks. Hence, conceptually, the relationship between the

natural path-based semantics of nonmonotonic inheritance networks and other more general nonmonotonic logics such as default logic, autoepistemic logic, etc. remains unclear. The goal of this paper is to address this problem. We do that by studying the relationship between the argumentation framework given in [2] and inheritance networks.

We show that each acyclic, consistent nonmonotonic inheritance network $\Gamma$ can be viewed as an argumentation framework $\mathrm{AF}_\Gamma$ so that the credulous semantics of $\Gamma$ "coincides" with the stable semantics of $\mathrm{AF}_\Gamma$. Further, we prove that the grounded semantics of $\mathrm{AF}_\Gamma$ is contained in the skeptical semantics of $\Gamma$ [13,25]. Thus, we can say that grounded semantics provides the baseline of skepticism in inheritance reasoning.

We present a modular transformation of consistent nonmonotonic networks into extended logic programs and show that the credulous semantics of the former coincides with the answer set semantics [8] of the latter. To our knowledge, this is the first transformation of nonmonotonic inheritance network into other more general nonmonotonic logics preserving the semantics of nonmontonic inheritance networks.

## 2 Preliminaries

### 2.1 Inheritance network

A defeasible inheritance network $\Gamma$ is defined here as a finite collection of positive and negative direct links between nodes. If x, y are nodes then $x \rightarrow y$ (resp. $x \not\rightarrow y$) represents a positive (resp. negative) *direct link* from x to y. A network $\Gamma$ is *consistent* if there exist no two nodes x, y such that both $x \rightarrow y$ and $x \not\rightarrow y$ belong to $\Gamma$. A *positive path* from $x_1$ to $x_n$ through $x_2$, ..., $x_{n-1}$, denoted by $\pi(x_1, \sigma, x_n)$ or $\pi(x_1, x_2, ..., x_{n-1}, x_n)$, is a sequence of direct links $x_1 \rightarrow x_2$, $x_2 \rightarrow x_3$, ... , $x_{n-1} \rightarrow x_n$. Similarly, a *negative path* from $x_1$ to $x_n$ through $x_2$, ..., $x_{n-1}$, denoted by $\bar{\pi}(x_1, \sigma, x_n)$ or $\bar{\pi}(x_1, x_2, ..., x_{n-1}, x_n)$, is a sequence of direct links $x_1 \rightarrow x_2$, $x_2 \rightarrow x_3$, ... , $x_{n-1} \not\rightarrow x_n$. A generalized path is a sequence of direct links $(x_1, x_2)$, $(x_2, x_3)$, ... , $(x_{n-1}, x_n)$, where $(x_i, x_{i+1})$ denotes a positive or negative direct link. $\Gamma$ is *acyclic* if there is no generalized path $(x_1, x_2)$, $(x_2, x_3)$, ... , $(x_{n-1}, x_n)$ with $x_1 = x_n$. The degree of the path $\alpha = \pi(x, \sigma, y)$ (resp. $\alpha = \bar{\pi}(x, \sigma, y)$), denoted by $deg_\Gamma(\alpha)$, is defined as the length (number of edges) of the longest generalized path from x to y. Furthermore, we also use the notation $\pi(x_1, \sigma, x_{n-1}) \rightarrow x_n$ (resp. $\pi(x_1, \sigma, x_{n-1}) \not\rightarrow x_n$) to denote the path $\pi(x_1, x_2, ..., x_{n-1}, x_n)$ (resp. $\bar{\pi}(x_1, x_2, ..., x_{n-1}, x_n)$).

¿From now on we will use $\Gamma$ to denote an arbitrary but fixed network and $\Phi$ to denote a set of paths in $\Gamma$ if no confusion is possible. The notion of inheritability presented here relies on three concepts: constructibility, conflict, and preemption. Their definitions are taken from [14,28,13,24,25].

**Definition 1.** A positive path $\pi(x, \sigma, u) \rightarrow y$ is *constructible* in $\Phi$ iff $\pi(x, \sigma, u) \in \Phi$ and $u \rightarrow y \in \Gamma$. A negative path $\pi(x, \sigma, u) \not\rightarrow y$ is constructible in $\Phi$ iff $\pi(x, \sigma, u) \in \Phi$ and $u \not\rightarrow y \in \Gamma$.

**Definition 2.** $\pi(x, \sigma, y)$ *conflicts* with any path of the form $\bar{\pi}(x, \tau, y)$ and vice versa. A path $\sigma$ is conflicted in $\Phi$ iff $\Phi$ contains a path that conflicts with $\sigma$.

Different ways have been proposed to define defeasible preemption in $\Phi$ [10, 14,13,27,25,24]. Here, we follow the *off-path* preemption given in [13].

**Definition 3.** A positive path $\pi(x, \sigma, u) \rightarrow y$ (see 1) is *preempted* in $\Phi$ iff there is a node v such that (i) $v \not\rightarrow y \in \Gamma$ and (ii) either v=x or there is a path of the form $\pi(x, \tau_1, v, \tau_2, u) \in \Phi$. Similarly, a negative path $\pi(x, \sigma, u) \not\rightarrow y$ is *preempted* in $\Phi$ iff there is a node v such that (i) $v \rightarrow y \in \Gamma$ and (ii) either v=x or there is a path of the form $\pi(x, \tau_1, v, \tau_2, u) \in \Phi$ .

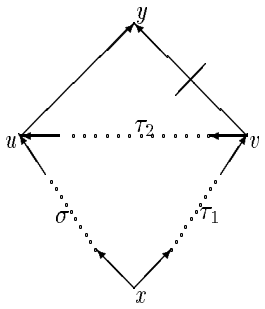



**Fig. 1.** $\pi(x, \sigma, u) \rightarrow y$ is preempted

The credulous semantics of an inheritance network is given in the following definition.

**Definition 4.** [13] $\sigma$ is *defeasibly inheritable* in $\Phi$, written as $\Phi \mid\sim \sigma$, iff
either $\sigma$ is a direct link
or $\sigma$ is a compound path, $\sigma$=$\pi(x, \tau, y)$ (likewise for negative path) such that
(i) $\sigma$ is constructible in $\Phi$, and
(ii) $\sigma$ is not conflicted in $\Phi$, and
(iii) $\sigma$ is not preempted in $\Phi$.

**Definition 5.** A set $\Phi$ of paths is a *credulous extension* of the net $\Gamma$ iff $\Phi = \{ \sigma : \Phi \mid\sim \sigma \}$.

### 2.2 Argumentation framework

In the following section, the basics of the abstract theory of argumentation framework of Dung [2] is recalled.

**Definition 6.** An *argumentation framework* is a pair $AF=<AR, attacks>$, where $AR$ is a set of arguments, and $attacks \in AR \times AR$.

If $(A, B) \in attacks$ we say A attacks B or B is attacked by A. A set of arguments S is said to be attacked by an argument A if there is $B \in S$ such that $(A, B) \in attacks$. Similarly, we say S attacks A if there is $B \in S$ such that $(B, A) \in attacks$.

**Definition 7.** A set of arguments S is said to be *conflict-free* if there exist no arguments A,B in S such that $(A, B) \in attacks$.

The stable semantics of AF is defined as follows.

**Definition 8.** A conflict-free set of arguments S is called a *stable extension* of AF if S attacks every argument which does not belong to S.

It is easy to see that the following proposition holds.

**Proposition.** *$S$ is stable iff $S$=$\{A \mid A$ is not attacked by $S\}$.*

The stable semantics of argumentation framework captures the semantics of many other mainstream approaches to nonmonotonic reasoning such as extension of Reiter's Default Logic [21], stable expansion of Autoepicstemic Logic [19], and stable model of Logic Programming [7]. We will see in section 3 that the credulous semantics of an inheritance network coincides with the stable semantics of a corresponding argumentation framework.

Often, a more restricted form of skeptical semantics is advocated in many approaches to nonmonotonic reasoning [20,31]. This form of skeptical semantics is defined in the argumentation framework by the notion of grounded extension defined as the least fixpoint of the following operator.

$F_{AF} : 2^{AR} \rightarrow 2^{AR}$, where

$F_{AF}$(S) = {A | $\forall$B, if B attacks A, then $\exists$C $\in S$ such that C attacks B}.

The idea behind this operator will become clear in the next definition.

**Definition 9.** An argument A is *defendable* wrt S iff for every argument B, if B attacks A, then there is an argument C in S such that C attacks B.

So, we can redefine $F_{AF}$ by $F_{AF}(S)$ = {A | A is defendable wrt S}.

The grounded extension of an argumentation framework is defined next.

**Definition 10.** The *grounded extension* of an argumentation framework AF denoted by $GE_{AF}$ is the least fixpoint of $F_{AF}$.

It has been pointed out in [2] that both the semantics of Pollock's Inductive Defeasible Logic [20], and well-founded semantics of Logic Programming [31] are captured by the grounded extension of argumentation framework.

The maximal fixpoint of $F_{AF}$ are called the preferred extension of AF. In general, stable extension are preferred extension but not vice versa. But as we will see later, for any argumentation framework corresponding to inheritance networks, stable semantics and preferred semantics coincide. So it is enough for us to work only with stable semantic.

## 3 Inheritance Networks as Argumentation Frameworks

Our goal is to clarify the conceptual relationship between the semantics of nonmonotonic inheritance networks and the semantics of argumentation frameworks. This will also help to illuminate the conceptual relationship between the semantics of nonmonotonic inheritance networks and other general approaches to nonmonotonic reasoning due to a result of Dung [2] showing that many general approaches to nonmonotonic reasoning [21,19,31] can be seen as special cases of argumentation frameworks.

We will show that every nonmonotonic inheritance network $\Gamma$ can be considered as an argumentation framework $AF_\Gamma = <AR_\Gamma, attacks_\Gamma>$ such that the credulous semantics of $\Gamma$ coincides with the stable semantics of $AF_\Gamma = <AR_\Gamma, attacks_\Gamma>$ in the sense that every credulous extension of $\Gamma$ is a stable extension of $AF_\Gamma$ and vice versa.

A path $\alpha = \pi(x, \sigma, u)$ is called a *prefix* of path $\beta = \pi(x, \sigma, u, \tau, v)$ in $\Gamma$. The set of all prefixes of $\beta$ is denoted by $pre(\beta)$. $\alpha$ is a *proper prefix* of $\beta$ iff $\alpha \in pre(\beta)$ and $\alpha \neq \beta$.

First, it is intuitive to view any path of $\Gamma$ as an argument. So, we have $AR_\Gamma = \{\sigma | \sigma \text{ is a path in } \Gamma\}$.

As next we define the attacks relation of $AF_\Gamma$. The underlying principle in defining the attacks relation is that more specific information overrides less specific one. For two conflicted paths, $\sigma$ and $\tau$, there are following cases:

(i) $\sigma$ is a direct link. It is clear that we should let $\sigma$ attack $\tau$, but not vice versa.

(ii) $\sigma$ and $\tau$ are compound paths. In this case, neither $\sigma$ nor $\tau$ are more specific than the other path. Thus, we have: $\sigma$ attacks $\tau$ and vice versa.

Further, it should be also clear that if $\sigma$ attacks a prefix $\alpha$ of $\tau$ then $\sigma$ attacks $\tau$.

We now consider another kind of attack.

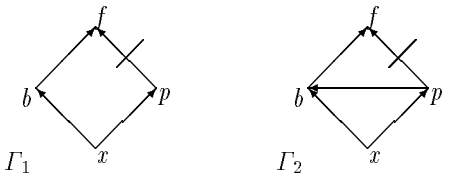



**Fig. 2.** Motivation of Attack Definition

*Example 1.* Consider the inheritance network $\Gamma_1$ in figure 2. The paths $\sigma = \pi(x, b, f)$ and $\tau = \bar{\pi}(x, p, f)$ are in conflict. Thus, $\pi(x, b, f)$ attacks $\bar{\pi}(x, p, f)$ and vice versa as in case (ii). So, $AF_{\Gamma_1}$=$<AR_{\Gamma_1}, attacks_{\Gamma_1}>$ with $attacks_{\Gamma_1} = \{(\sigma, \tau), (\tau, \sigma)\}$. Hence $AF_{\Gamma_1}$ has two stable extensions corresponding to two credulous extensions of $\Gamma_1$ : $E_1 = \Gamma \cup \{\sigma\}$ and $E_2 = \Gamma \cup \{\tau\}$.

The network $\Gamma_2$ in figure 2 is obtained from the network $\Gamma_1$ by adding the positive link $p \rightarrow b$. This is the well- known Penguin-Bird-Fly example. It is clear that in $AF_{\Gamma_2}$=$<AR_{\Gamma_2}, attacks_{\Gamma_2}>$, $\sigma$ attacks $\tau$ and $\tau$ attacks $\sigma$ as in case of $\Gamma_1$. Adding the link $p \rightarrow b$ into $\Gamma_1$ makes the argument $\tau$ more specific than the argument $\sigma$. Thus, due to the principle that more specific information can override less specific one, we can say that adding $p \rightarrow b$ to $\Gamma_1$ creates an attack of new kind against $\sigma = \pi(x, b, f)$. We can represent this by viewing the argument $\alpha = \pi(x, p, b)$ in the presence of the link $p \not\rightarrow f$ as an attack against the path $\sigma = \pi(x, b, f)$.

These motivations lead to the following definition of attacks.

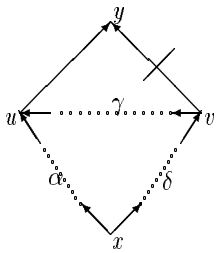



**Fig. 3.** $\pi(x, \alpha, u) \rightarrow y$ is attacked by $\pi(x, \delta, v, \gamma, u)$ in presence of $v \not\rightarrow y$

**Definition 11.** A path $\sigma$ attacks path $\tau$ iff

(a) $\sigma$ is a direct link x$\rightarrow$ $y$ (resp. $x \not\rightarrow y$) and $\tau = \bar{\pi}(x, \alpha, y)$ (resp. $\tau = \pi(x, \alpha, y)$) or

(b) $\sigma$ is in conflict with some compound path $\delta \in pre(\tau)$ or

(c) $\sigma, \tau$ are compound paths where there exists a prefix $\beta = \pi(x, \alpha, u) \rightarrow y$ of $\tau$ (resp. $\pi(x, \alpha, u) \not\rightarrow y$) and $\sigma = \pi(x, \delta, v, \gamma, u)$ with $v \not\rightarrow y \in \Gamma$ (resp. $v \rightarrow y \in \Gamma$) (see figure 3).

*Remark.* From now on we will refer to three types of attacks (a), (b), and (c) defined above as attack by direct link, by conflict, and by preemption, respectively.

So, in our point of view there are two kinds of attacks between two paths $\sigma$ and $\tau$, symmetric ($\tau$ attacks $\sigma$ and $\sigma$ attacks $\tau$) and asymmetric ($\tau$ attacks $\sigma$ but not vice versa). Symmetric attacks are equivalent to conflictor in [28] while asymmetric attacks have some similarity to preemptor in [28] but not identical.

*Example 2.* (Continuation of example 1) For $\Gamma_2$ in figure 2 we have $AF_{\Gamma_2}$=$<AR_{\Gamma_2}, attacks_{\Gamma_2}>$ with

$AR_{\Gamma_2} = \Gamma_2 \cup \{\sigma, \tau, \alpha, \beta, \delta\}$ and

$attacks_{\Gamma_2} = \{(p \not\rightarrow f, \delta)\} \cup \{(\sigma, \tau), (\tau, \sigma), (\tau, \beta), (\beta, \tau)\} \cup \{(\alpha, \sigma), (\alpha, \beta)\}$

where $\beta = \pi(x, p, b, f)$, and $\delta = \pi(p, b, f)$. Here, $\{(p \not\rightarrow f, \delta)\}$ is the set of attacks by direct link, $\{(\sigma, \tau), (\tau, \sigma), (\tau, \beta), (\beta, \tau)\}$ is the set of attacks by conflict, and $\{(\alpha, \sigma), (\alpha, \beta)\}$ is the set of attacks by preemption. □

We now prove the coincidence between the credulous extension of $\Gamma$ and the stable extension of $AF_\Gamma$.

**Theorem 12.** *Let $E$ be a set of paths in $\Gamma$. Then, $E$ is a stable extension of $AF_\Gamma$ iff $E$ is a credulous extension of $\Gamma$.*

*Proof.* See Appendix. □

The relationship between grounded semantics of $AF_\Gamma$ and skeptical semantics of $\Gamma$ is discussed in the section 5.

## 4 Transforming Inheritance Network into Logic Program

The coincidence between the credulous semantics of an inheritance network $\Gamma$ and the stable semantics of the corresponding argumentation framework $AF_\Gamma$ together with the results in [2] stating that argumentation frameworks in principle

can be represented as logic programs, points out that an inheritance network $\Gamma$ can be transformed into an equivalent logic program $P_\Gamma$. Thus, proof procedures based on negation-as-failure can be applied to $P_\Gamma$ to compute the credulous semantics of $\Gamma$.

In this section we give a direct and modular transformation an inheritance network $\Gamma$ into an extended logic program $P_\Gamma$ and show that the credulous semantics of $\Gamma$ coincides with the answer set semantics of $P_\Gamma$[1].

In following we assume that the readers are family with the answer set semantics of Logic Programs [8].

The set of nodes of $\Gamma$ is the union of two disjoint sets, the set of individuals of $\Gamma$, denotes by $I_\Gamma$, consists of all nodes x of $\Gamma$ such that there exists no direct link $y \rightarrow x$ or $y \not\rightarrow x$ in $\Gamma$, and the set of predicate (or properties) nodes. In following, a, b, c, ... will represent the individuals of $\Gamma$ and p, q, r, ... are the predicate nodes if not otherwise specified. For example, in the example 3 below, a denotes the individual Nixon, and p, r and q denote the predicates Pacifist, Republican and Quaker, respectively.

We first assign an unique natural number $j \in N$ to each direct link $p \rightarrow q \in \Gamma$ (resp. $p \not\rightarrow q \in \Gamma$) of $\Gamma$, $p \notin I_\Gamma$, written as $p \rightarrow_j q$ (resp. $p \not\rightarrow_j q$), and introduce a new predicate $ab_j$ representing the abnormal-literal at the edge j. The link $p \rightarrow_j q$ (or $p \not\rightarrow_j q$) is then referred simply as the link j. Based on the attack relationship of $AF_\Gamma$ the inheritance network $\Gamma$ can be transformed into an extended logic program $P_\Gamma$ as follows.

Any direct link $a \rightarrow p$ (resp. $a \not\rightarrow p$) beginning from an individual node a can be transformed directly into a fact of $P_\Gamma$ because of there is no arguments which attack $a \rightarrow p$ (resp. $a \not\rightarrow p$). Hence, we have:

1. For each $a \in I_\Gamma$ if $a \rightarrow p$ (resp. $a \not\rightarrow p$) is in $\Gamma$ then

   $$p(a) \leftarrow \text{ (resp. } \neg p(a) \leftarrow)$$

   is a clause of $P_\Gamma$.

2. For $p \notin I_\Gamma$ and each direct link $p \rightarrow_j q \in \Gamma$, the two clauses

   $$\begin{array}{ll} q(x) & \leftarrow p(x),\ not\ ab_j(x) \quad \text{and} \\ ab_j(x) & \leftarrow \neg q(x) \end{array}$$

   belong to $P_\Gamma$.

[1] Grégoire [10] presented an algorithm for transformation of an inheritance network into a logic program. But in our view this could not be considered as a proper transformation because according to this algorithm we first have to compute the extensions of the network and then define a logic program specifying this extension.

Similarly,

3. For $p \notin I_\Gamma$ and each direct link $p \not\rightarrow_j q \in \Gamma$

$$\neg q(x) \leftarrow p(x),\ not\ ab_j(x) \quad \text{and}$$
$$ab_j(x) \leftarrow q(x)$$

are clauses of $P_\Gamma$.

4. For each pair of direct links $p \rightarrow_j q$ and $r \not\rightarrow_k q$ in $\Gamma$
   (a) If there exists a positive path from p to r over the links $j_1, ..., j_n$ then the clause

$$ab_k(x) \leftarrow p(x),\ not\ ab_{j_1}(x), \ldots,\ not\ ab_{j_n}(x)$$

   belongs to $P_\Gamma$.
   (b) If there exists a positive path from r to p over the links $j_1, ..., j_n$ then the clause

$$ab_j(x) \leftarrow r(x),\ not\ ab_{j_1}(x), \ldots,\ not\ ab_{j_n}(x)$$

   belongs to $P_\Gamma$.

We illustrate the transformation from $\Gamma$ into $P_\Gamma$ in the next two examples.

*Example 3.* (Nixon-Diamond)

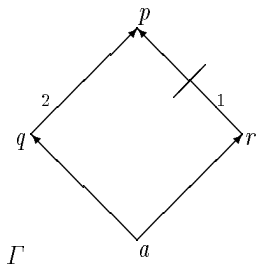



**Fig. 4.** a=Nixon, p=Pacifist, q=Quaker, r=Republican

The corresponding program $P_\Gamma$ of $\Gamma$ in the figure 4 is:

| | |
|---|---|
| $a \rightarrow r$ | $r(a) \leftarrow$ |
| $a \rightarrow q$ | $q(a) \leftarrow$ |
| $r \not\rightarrow_1 p$ | $\neg p(x) \leftarrow r(x),\ not\ ab_1(x)$ |
| | $ab_1(x) \leftarrow p(x)$ |
| $q \rightarrow_2 p$ | $p(x) \leftarrow q(x),\ not\ ab_2(x)$ |
| | $ab_2(x) \leftarrow \neg p(x)$ |

It is easy to see that $P_\Gamma$ has two answer sets $\{r(a), q(a), p(a), ab_1(a)\}$ and $\{r(a), q(a), \neg p(a), ab_2(a)\}$. □

*Example 4.* (Penguin-Bird-Fly)

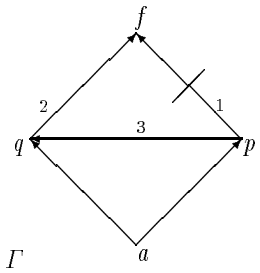



**Fig. 5.** a=Tweety, p=Penguin, q=Bird, f=Fly

Let consider the net $\Gamma$ in figure 5. The corresponding program $P_\Gamma$ consists of

| | |
|---|---|
| $a \rightarrow p$ | $p(a) \leftarrow$ |
| $a \rightarrow q$ | $q(a) \leftarrow$ |
| $p \not\rightarrow_1 f$ | $\neg f(x) \leftarrow p(x),\ not\ ab_1(x)$ |
| | $ab_1(x) \leftarrow f(x)$ |
| $q \rightarrow_2 f$ | $f(x) \leftarrow q(x),\ not\ ab_2(x)$ |
| | $ab_2(x) \leftarrow \neg f(x)$ |
| $p \rightarrow_3 q$ | $q(x) \leftarrow p(x),\ not\ ab_3(x)$ |
| | $ab_3(x) \leftarrow \neg q(x)$ |
| and | $ab_2(x) \leftarrow p(x),\ not\ ab_3(x)$ |

The unique answer set of $P_\Gamma$ is $\{p(a), q(a), \neg f(a), ab_1(a)\}$. □

We next discuss the relationship between the answer set semantics of $P_\Gamma$ and the credulous semantics of $\Gamma$.

First, for any set of paths E in $\Gamma$ we define:

$$M_E = \{q(a) | \exists \pi(a, \ldots, q) \in E\} \cup \{\neg q(a) | \exists \overline{\pi}(a, \ldots, q) \in E\}$$

Further, let AB denotes the set of all grounded instance of abnormal-predicates in $P_\Gamma$, that is,

$$AB_\Gamma = \{ab_j(a) | a \in I_\Gamma, j \text{ is a rule link in } \Gamma\}.$$

The next theorem point out the relationship between the answer set semantics of the program $P_\Gamma$ and the credulous semantics of the inheritance network $\Gamma$.

**Theorem 13.** *$M$ is an answer set of $P_\Gamma$ iff there exists a credulous extension $E$ of $\Gamma$ such that $M_E = M \backslash AB_\Gamma$.*

*Proof.* See appendix. □

In the subsequent section we denote the transformation from $\Gamma$ into $P_\Gamma$ by $T$, that is, $T(\Gamma) = P_\Gamma$ and study the modularity of the transformation $T$. In [13] Horty suggests that a transformation $T$ is modular if $T(\Gamma \cup \Delta) = T(\Gamma) \cup T(\Delta)$. We do agree partly with Horty that introducing new links to $\Gamma$ should not force us to withdraw any results of the previous transformation, that is, $T(\Gamma) \subseteq T(\Gamma \cup \Delta)$. However, adding links to an inheritance network can establsih *new relationships* among the links in $\Delta$ and $\Gamma$ which do not occure in $\Delta$ and $\Gamma$. These newly established relationships should be taken into account in the transformation process. Our improved definition of modularity is as follows.

**Definition 14.** A transformation T is modular if for each eligible extension[2] $\Gamma \cup \Delta$ of $\Gamma$

$$T(\Gamma \cup \Delta) = T(\Gamma) \cup T(\Delta, \Gamma)$$

where $T(\Gamma)$ represent the result of the transformation of $\Gamma$ and $T(\Delta, \Gamma)$ is the result of the transformation of $\Delta$ under the "context" $\Gamma$.

Obviously we can divide links in $\Gamma$ into two types: *factual link* which begins from individual node (e.g. $a \rightarrow p$, $a \not\rightarrow p$) and *rule link* which begins from predicate node (e.g. $p \rightarrow q$, $p \not\rightarrow q$). An extension of an inheritance network $\Gamma$ is said to be *eligible* if it does not change the concepts of $\Gamma$, i.e., none of the factual links will be changed to rule link after the extension. The next example helps to clarify that point.

*Example 5.* Consider the following networks:

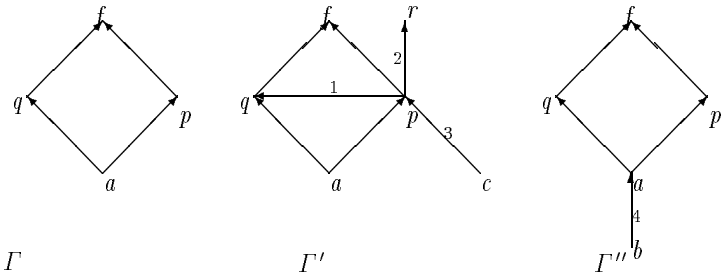


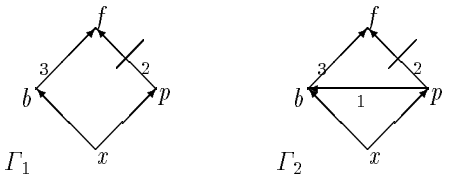

**Fig. 6.** Adding new links to an inheritance networks

In the above figure, adding links 1, 2, or 3 to $\Gamma$ ($\Gamma'$, figure 6) is an eligible extension but adding link 4 to $\Gamma$ ($\Gamma''$, figure 6) is not an eligible extension because it changes the factual link $a \rightarrow p$ (likewise $a \rightarrow q$) into a rule link. □

[2] The definition of eligible extension is given in definition 15.

**Definition 15.** An extension $\Gamma \cup \Delta$ to an inheritance network $\Gamma$ is *eligible* if it does not change any factual rule of $\Gamma$ into a rule link in $\Gamma'$.

In following we will discuss the modularity of the transformation $T$ only with respect to eligible extensions of $\Gamma$. The next example will illustrate how $T(\Delta, \Gamma)$ can be determined.




Consider the two networks $\Gamma_1$ and $\Gamma_2$ in the figure above. Given that $T(\Gamma) = P_\Gamma$. The transformation of $\Gamma_1$ and $\Gamma_2$ into logic programm has been shown in the examples 3 and 4. Let $\Delta = \Gamma_2 \backslash \Gamma_1 = \{p \rightarrow_1 b\}$. Then, it is clear that

$$P_{\Gamma_2} = P_{\Gamma_1} \cup P_{(\Delta, \Gamma_1)}$$

with $P_{(\Delta,\Gamma_1)}$ consists of
$$q(x) \leftarrow p(x), not\ ab_1(x), \text{ and}$$
$$ab_1(x) \leftarrow \neg q(x), \text{ and}$$
$$ab_3(x) \leftarrow p(x), not\ ab_1(x).$$

Given are $\Delta$ and $\Gamma$. We now show how $T(\Delta, \Gamma)$ can be determined. First, we introduce the notion of "specificity triangle" as follows.

**Definition 16.** A set of links in $\Gamma$ is called a positive (or negative) *specificity triangle* if and only if E consists of two opposite links $p \rightarrow q$ and $r \not\rightarrow q$ and the links belonging to a positive path from p to r (resp. from r to p) and no other links of $\Gamma$ belong to E.

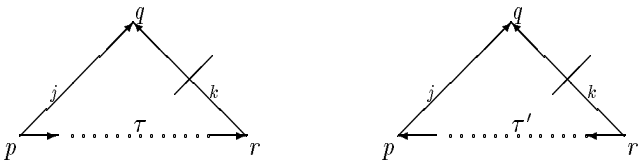



**Fig. 7.** positive specificity triangle (E,p,k,j) and negative specificity triangle (E,r,j,k)

In following we will denote a specificity triangle E of $\Gamma$ containing $p \rightarrow_j q$, $r \not\rightarrow_k q$ and a positive path from p to r (resp. from r to p) by (E,p,k,j) (resp. (E,r,j,k)) (figure 7).

$T(\Delta, \Gamma)$ is defined recursively as follows.

$$T(\Delta, \Gamma) = T(\{\alpha\}, \Gamma) \cup T(\Delta \backslash \{\alpha\}, \Gamma \cup \{\alpha\})$$

where $\alpha$ is a link in $\Delta$ and $T(\{\alpha\}, \Gamma)$ is defined by $T(\{\alpha\}, \Gamma) = T_1 \cup T_2$ with

1. Case: $\alpha$ is a factual link, i.e., p is an individual a. Then,

   - $\alpha$ is positive: $T_1 = \{q(a) \leftarrow\}$, $T_2 = \emptyset$.

   - $\alpha$ is negative: $T_1 = \{\neg q(a) \leftarrow\}$, $T_2 = \emptyset$.

2. Case: $\alpha$ is a rule link, i.e., p is a predicate. Then,

   - $\alpha$ is positive: $T_1 = \{q(x) \leftarrow p(x),\ not\ ab_j(x)\} \cup \{ab_j(x) \leftarrow \neg q(x)\}$.

   - $\alpha$ is negative: $T_1 = \{\neg q(x) \leftarrow p(x), not\ ab_j(x)\} \cup \{ab_j(x) \leftarrow q(x)\}$.

   - and

     $$T_2 = \{\ ab_k(x) \leftarrow p(x),\ not\ ab_{j_1}(x), \ldots,\ not\ ab_{j_n}(x) \mid \exists\ a\ specificity\ triangle\ (E, p, k, t),\ \alpha \in E,\ E \backslash \{k, t\} = \{j_1, \ldots, j_n\}\}$$

Note that in both cases $T_1$ comprises of clauses that are directly transformed from the link $\alpha$ in $\Gamma \cup \{\alpha\}$ while $T_2$ contain clauses which represent the change of $\Gamma$ in $T(\Gamma \cup \{\alpha\})$ if $\alpha$ is added to $\Gamma$.

The next theorem follows directly from the above equations.

**Theorem 17.** *$T$ is modular.*

*Proof.* Obvious by induction.

## 5 Skeptical semantics of inheritance network

### 5.1 Grounded skeptical semantics of inheritance networks

The skeptical semantics for inheritance network is defined by the notion of ideally skeptical extension and is defined as follows.

**Definition 18.** [25,24] The intersection of all credulous extensions of $\Gamma$ is called the *ideally skeptical extension* of $\Gamma$.

We give now the definition of the grounded skeptical semantics for an inheritance network.

**Definition 19.** The grounded skeptical semantics of the inheritance network $\Gamma$ is defined as the grounded extension $GE_{AF_\Gamma}$ of the corresponding argumentation framework $AF_\Gamma$ of $\Gamma$.

Since $GE_{AF_\Gamma}$ is contained in every stable extension of $AF_\Gamma$ we have the following theorem.

**Theorem 20.** *The grounded extension $GE_{AF_\Gamma}$ of $AF_\Gamma$ is a subset of the ideally skeptical extension of $\Gamma$.*

*Proof.* Since $GE_{AF_\Gamma}$ is contained in every stable extension it is contained in the intersection of all stable extension of $AF_\Gamma$ and therefore $GE_{AF_\Gamma}$ is contained in their intersection which is the ideally skeptical extension. □

Grounded skeptical semantics can be computed by using SLDNF- resolution in $P_\Gamma$. We next discuss another kind of skeptical semantics of inheritance network and its relationship to grounded skeptical semantics.

### 5.2 Iterative skeptical semantics and grounded skeptical semantics

Another sort of skeptical semantics of nonmonotonic inheritance networks is given in [13]. This semantics is called an iterative skeptical semantics due to the fact that it is defined as the result of an iterative process.

After recalling the definition of iterative skeptical semantics from [13], we will show that the grounded skeptical semantics is more skeptical than the iterative skeptical semantic.

**Definition 21.** [13] A path $\sigma$ is permitted in $\Phi$, denoted by $\Phi \mid\!\approx \sigma$, iff either

(i) If $\sigma$ is a direct link then $\Phi \mid\!\approx \sigma$ iff $\sigma \in \Phi$ or

(ii) If $\sigma$ is a compound path then $\Phi \mid\!\approx \sigma$ iff $\Phi \mid\sim \sigma$, and $\nexists\, \tau$ such that $\Phi \mid\sim \tau$ and $\tau$ is in conflict with $\sigma$.

**Definition 22.** [13] The sequence of path sets $\Phi_1, \Phi_2, \ldots$ is given by

$\Phi_1 = \Gamma$, and

$\Phi_{n+1} = \Phi_n \cup \{\sigma | deg_\Gamma(\sigma) = n+1 \text{ and } \Phi_n \mid\!\approx \sigma\}$.

The paths set $HS = \bigcup_{n=1}^{\infty} \Phi_n$ is called the skeptical extension of $\Gamma$.

The iterative skeptical semantics in [13] represents the idea of ambiguity blocking in nonmonotonic inheritance networks. We will show next that this is basically reasoning in two levels, first with the asymmetric attacks and afterward with the symmetric attacks. In the first level we try to determine the possible defendable paths by taking into consideration only asymmetric attacks of $AF_\Gamma$. In the second level we will discard all the paths which are conflicted. We define a special subset of attacks which consists of all asymmetric attacks of $attacks_\Gamma$ as follows.

$$g\text{-}attacks = \{(\sigma, \gamma) \in attacks_\Gamma \mid (\gamma, \sigma) \notin attacks_\Gamma\}.$$

It is easy to see that

$$g\text{-}attacks = attacks_\Gamma \backslash \{(\sigma, \tau) \in attacks_\Gamma \mid \sigma \text{ and } \tau \text{ are conflicted paths}, \sigma \notin \Gamma\}.$$

The next example illustriates these two kinds of attack.

*Example 6.* (g-attacks and attacks)

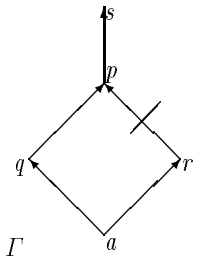



**Fig. 8.** g-attacks and attacks

In the figure above we have $a \rightarrow r \not\rightarrow p$ attacks $a \rightarrow q \rightarrow p \rightarrow s$ and it is also a g-attacks. And, the two paths $a \rightarrow r \not\rightarrow p$ and $a \rightarrow q \rightarrow p$ attacks each other but none of these attacks is a g-attacks.

Let $\Gamma$ be an consistent, acyclic network and $AF_\Gamma$=<$AR_\Gamma, attacks_\Gamma$> be the corresponding argumentation framework of $\Gamma$. We define $AF_0$=<$AR_\Gamma$,*g-attacks*>. Further, let $GE_0$ be the grounded extension of $AF_0$. The argumentation framework $AF_1$ is defined by $AF_1$=<$GE_0, att$> where att is the restriction of attacks$_\Gamma$\ g-attacks on $GE_0$. Let $GE_1$ be the grounded extension of $AF_1$. We will show that $GE_1$ coincides with the skeptical extension HS of $\Gamma$.

The follow example shows how $GE_1$ captures the ambiguity blocking idea in HS.

*Example 7.* (Ambiguiity Blocking and Argumentation)

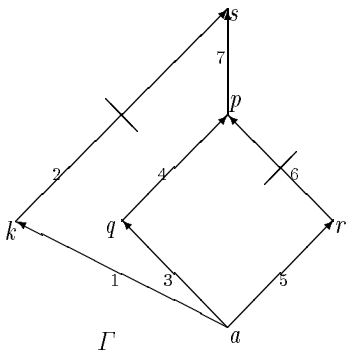



**Fig. 9.** Ambiguity Blocking and Argumentation

In figure 9, let denote the paths by the sequence of links, e.g., $\bar{\pi}(a,k,s)$ =12, $\pi(a,q,p,s)$ =347, etc. Then, $HS = \Gamma \cup \{12, 47\}$ and g-attacks=$\{56, 347\}$. By

definition, $AF_0 = (AR_\Gamma$,g-attacks). It follows immediately that $AF_1 = (\Gamma \cup \{56, 34, 12, 47\}, \{(56, 34), (34, 56)\})$. ¿From that it is easy to see that $GE_1 = HS = \Gamma \cup \{12, 47\}$. □

In the above example we can see that in the first step we eliminate the argument 347 ($a \rightarrow q \rightarrow p \rightarrow s$) because it is g-attacked by 56 ($a \rightarrow r \not\rightarrow p$) and none of the available argument g-attacks 56. This represents the idea that the ambiguity at the node p blocks the path 347 from acceptable. In the second steps we eliminate both two arguments 34 ($a \rightarrow r \not\rightarrow p$) and 56 ($a \rightarrow q \rightarrow p$) to resolve the ambiguity at node p. It is worth to notice that while Horty resolve the ambiguity at node p first before continueing at node s our approach is in the reverse order. However, the result of both approaches are the same as the next theorem pointed out.

**Theorem 23.** *The iterative skeptical extension HS coincides with the grounded extension $GE_1$ of the alternative argumentation framework $AF_1$.*

*Proof.* See Appendix. □

Further, from the monotonicity of the characteristic function $F_{AF}$ [2] and the fact that *g-attacks* is a subset of $attacks_\Gamma$ we can easily prove that $GE_{AF_\Gamma} \subseteq GE_0$. This leads to the next theorem.

**Theorem 24.** *The grounded skeptical semantics $GE_{AF_\Gamma}$ of the inheritance network $\Gamma$ is contained in the iterative skeptical semantics HS of $\Gamma$.*

*Proof.* See Appendix. □

## 6 Conclusion

We have studied the relationship between semantical concepts of nonmonotonic inheritance and of argumentation framework. In section 3 we showed that argumentation framework, a general form of argument-based reasoning can be applied to specify nonmonotonic inheritance reasoning in a simple way. We proved that a consistent and acyclic network can be viewed as an argumentation framework so that the credulous semantics of the former coincides with the stable semantics of the latter. Furthermore, grounded semantics of argumentation framework can be seen as an approximation for skeptical semantics of inheritance reasoning.

The capturing of credulous semantics of nonmonotonic inheritance by argumentation framework shows that argumentation can be applied successfully to formulate nonmonotonic inheritance reasoning. It is interesting to note that many newly developed approaches to nonmonotonic reasoning [6,23,1] can not applied directly to inheritance reasoning as we did for Dung's argumentation framework. Geffner and Pearl's conditional entailment [6] is too weak as we can not draw the conclusion "Tweety is an animal with feather" if we replace

the rule "Bird fly" by two rules "Bird are animals with feather" and "Animal with feather fly" in the Penguin-Bird-Fly example. Simary and Loui's defeasible reasoning [23] gives unintuitive answer even in simple cases as in the Penguin-Bird-Fly example given that Tweety is a penguin and a bird. Delgrande and Schaub's general approach [1] cannot give proper answer in the example with $R = \{a \rightarrow r, a \rightarrow p, a \rightarrow \neg q, p \rightarrow q, q \rightarrow \neg s, q \rightarrow r, r \rightarrow s\}$ (figure 18, page 158 [13]).

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## Appendix: Proof of selected theorems

### Section 3

In this section we prove that the credulous semantics of $\Gamma$ and the stable semantics of the corresponding argumentation framework coincide. At first, we prove some general properties of $AF_\Gamma$.

**Lemma 25.** *Let $\Gamma$ be an acyclic, consistent inheritance network and $AF_\Gamma$=<$AR_\Gamma, attacks_\Gamma$> is the corresponding argumentation framework. Then, if $\sigma$ attacks $\sigma'$ then $deg_\Gamma(\sigma) \leq deg_\Gamma(\sigma')$.*

*Proof.* Consider three cases:

1. $\sigma$ attacks $\sigma'$ by a direct link then it is clear that $deg_\Gamma(\sigma) = deg_\Gamma(\sigma')$ because $\sigma'$ and $\sigma$ have the same begin and the same end nodes.
2. $\sigma$ attacks $\sigma'$ by conflict. Then, either both $\sigma$ and $\sigma'$ have the same begin and end and therefore $deg_\Gamma(\sigma) \leq deg_\Gamma(\sigma')$ or there is some prefix of $\sigma'$ which is conflict with $\sigma$, in this case we have $deg_\Gamma(\sigma) < deg_\Gamma(\sigma')$.
3. $\sigma$ attacks $\sigma'$ by preemption. Then, there is a prefix $\alpha = \pi(x, \tau, u) \rightarrow y$ (resp. $\pi(x, \tau, u) \not\rightarrow y$) of $\sigma'$ such that $\sigma = \pi(x, \delta, v, \gamma, u)$ with $v \rightarrow y$ (resp. $v \not\rightarrow y$) in $\Gamma$. By definition of $deg_\Gamma$ we have $deg_\Gamma(\sigma) < deg_\Gamma(\sigma')$.

The lemma is proved from these three cases. □

**Lemma 26.** *Let $S$ be set of arguments and $\sigma$ be an argument defendable wrt $S$. Then, each $\alpha \in pre(\sigma)$ is defendable wrt $S$.*

*Proof.* If $\alpha \in pre(\sigma)$ and $\beta$ is an argument which attacks $\alpha$ then $\beta$ attacks $\sigma$ and therefore $\beta$ is attacked by S. Hence, $\alpha$ is defendable wrt S. □

**Lemma 27.** *Let $S$ be a set of argument in $AF_\Gamma$. If $S \mid\sim \sigma$ then $\sigma$ is not attacked by $S$.*

*Proof.* Assume that there exists an argument $\tau$ in S such that $(\tau, \sigma) \in attacks$. By definition of attack we have three cases:

1. $\tau$ attacks $\sigma$ by direct link. Then, $\sigma$ is preempted in S.
2. $\tau$ attacks $\sigma$ by conflict. Then, $\sigma$ is conflicted or non constructible in S.
3. $\tau$ attacks $\sigma$ by preemption. Then, $\sigma$ is preempted or non constructible in S. From the three cases, we learn that if $(\tau, \sigma) \in attacks$ then $S \not\mid\sim \sigma$. Contradictory !!! Thus, S does not attack $\sigma$. □

The next lemma follows directly from lemma 27, and the definition of the stable extension.

**Lemma 28.** *If $E$ is a stable extension of $AF_\Gamma$ and $\sigma \notin E$. Then, $E \not\mid\sim \sigma$.* □

Further, it is easy to prove the next two lemmas.

**Lemma 29.** *If $E$ is a stable extension of $AF_\Gamma$, then $\Gamma \subset E$, and for every path $\sigma \in E$, $\sigma$ is not preempted in $E$.* □

**Lemma 30.** *Let $E$ be a credulous extension of $\Gamma$. Then $\sigma$ is attacked by $E$ if $\sigma \notin E$ .*

*Proof.* Obviously, $\sigma$ is a compound path. Without a lost of generality, assume that $\sigma$ is a positive path $\pi(x, \tau, u) \rightarrow y$. Since $\sigma$ does not belong to E, $E \not\models \sigma$. Hence,

1. $\sigma$ is conflict in E. It means, there is some path $\alpha$ in E that is conflict with $\sigma$. Therefore, $\sigma$ is attacked by E.
2. $\sigma$ is not constructible in E. There is a $\alpha \in pre(\sigma)$ such that all proper prefix of $\alpha$ is contained in E but not $\alpha$. Therefore $E \not\models \alpha$. Further, since $\alpha$ is constructible in E. For we have two sub-cases:
   $\alpha$ is conflicted in E. Similarly to the first case, $\alpha$ is attacked by E. Thus, $\sigma$ is attacked by E.
   $\alpha$ is preempted in E. See next case.
3. $\sigma$ is preempted in E. That is, there is some node v with $v \not\rightarrow y \in \Gamma$ and there is some $\delta = \pi(x, \alpha, v, \beta, u)$ in E. Thus, $\delta$ attacks $\sigma$.

¿From the three cases, we can conclude that $\sigma$ is attacked by E. □

From the lemmae 25–30 we can prove the theorem 12:

**Proof of Theorem 12**

*Proof.* 1. '$\longleftarrow$' Suppose that E is a stable extension of $AF_\Gamma$. Let $\sigma \notin E$. ¿From lemma 27 we have $E \not\models \sigma$ (i). Now, let $\sigma \in E$. Any prefix of $\sigma$ is defendable wrt E (Lemma 26). Thus, all prefixes of $\sigma$ are containing in E. So, $\sigma$ is constructible and non-conflicted in E. Further, $\sigma$ is not preempted in E (Lemma 29). Hence, $\sigma$ is defeasibly inheritable in E (ii). From (i) and (ii) we can conclude that E is a credulous extensions of $\Gamma$.

2. '$\longrightarrow$' Now, if E is a credulous extension of $\Gamma$ then E is conflict-free (Lemma 26). It is easy to see that E attacks every argument which does not belong to E (Lemma 30). Hence, E is a stable extension of $AF_\Gamma$.

The theorem is proved. □

## Section 4

For a credulous extension E of $\Gamma$ let

$$Ab_E = \{ab_j(a) | a \in I_\Gamma, p \rightarrow_j q \in E, \exists \pi(a, \ldots, p) \in E,\ \nexists \pi(a, \ldots, q) \in E \ \ or \\ p \not\rightarrow_j q \in E, \exists \pi(a, \ldots, p) \in E,\ \nexists \bar{\pi}(a, \ldots, q) \in E\}$$

We prove that

**Lemma 31.** *Let E be a grounded creduloud extension of $\Gamma$. Then, $S = M_E \cup Ab_E$ is an answer set of $P_\Gamma$.*

*Proof.* S is an answer set of $P_\Gamma$ [8] if S is the answer set of $P_\Gamma(S)$ which is obtained by (i) deleting all clauses containing one literal *not* $ab_j(a)$ if $ab_j(a) \in S$ and (ii) deleting all the abnormal-literals in the remaining clauses. In following let $\alpha(P_\Gamma(S))$ be the answer set of $P_\Gamma(S)$. From the definition of $M_E$ and $Ab_E$, we have three cases:

1. Case

$q(a) \in S$ iff there is a positive path $\sigma$ from a to q in E
iff $ab_j(a) \in Ab_E$ for all link j belongs to $\sigma$
iff $ab_j(a) \notin S$ for all link j belongs to $\sigma$
iff $P_\Gamma(S)$ consists of $q_j(x) \leftarrow p_j(x)$ for all link j belongs to $\sigma$
iff $q(a) \in \alpha(P_\Gamma(S))$.

2. Case

$\neg q(a) \in S$ iff there is a negative path $\sigma$ from a to q
over the link $j_1, \ldots, j_n$ in E
iff $ab_j(a) \in Ab_E$ for all link j belongs to $\sigma$
iff $ab_j(a) \notin S$ for $j \leq n$ belongs to $\sigma$
iff $P_\Gamma(S)$ contains $\neg q_n(x) \leftarrow p_n(x)$
and $q_j(a) \leftarrow p_j(a)$ for $j < n$ belongs to $\sigma$ where $q_n$=q
iff $\neg q(a) \in \alpha(P_\Gamma(S))$.

3. Case

$ab_j(a) \in S$ iff $ab_j(a) \in Ab_E$ (Assume that the link j is $p \rightarrow q$)
iff there is a positive path $\pi(a, \sigma, p) \in E$ but $\tau = \pi(a, \sigma, p, q) \notin E$
iff the path $\tau$ is conflicted or preempted in E
iff $ab_j(x) \leftarrow \neg q(x)$ is a clause of $P_\Gamma(S)$ and $\neg q(a) \in S$
or $ab_j(x) \leftarrow r(x)$ is a clause of $P_\Gamma(S)$ and $r(a) \in S$
iff $ab_j(a) \in \alpha(P_\Gamma(S))$.

So, we have proved that S is an answer set of $P_\Gamma$.

It is easy to see that if $E$ is a credulous extension of $\Gamma$ then $S = M_E \cup Ab_E$ is a consistent answer set of $P_\Gamma$. Since credulous extension of inheritance networks always exists the following corollary yields directly from the proposition 3.1 in [8] which stated that a program cannot have two answer sets of which one is the proper subset of the other.

**Corollary 32.** *The program $P_\Gamma$ has only consistent answer sets.*

For a set of ground literal M of the programm $P_\Gamma$ we define
$E_M = \{\sigma \mid \sigma$ is a path in $\Gamma$ beginning from $a \in I_\Gamma, ab_j(a) \notin M$ for all link j on $\sigma\} \cup \{\sigma \mid \sigma$ is a link beginning from $a \in I_\Gamma\}$.

**Definition 33.** Let E be a credulous extension of $\Gamma$. Then, the set of paths in E beginning from the individual nodes of $\Gamma$, denoted by $E_G$, is called the *grounded kernel* of E.

Note that the grounded kernel $E_G$ can be seen as the extension focus on individual nodes of $\Gamma$. $E_G$ posseses also the maximality property of E and any path $\sigma \in E_G$ is defeasibly inheritable in $E_G$. We show that

**Lemma 34.** *Let $M$ be an answer set of $P_\Gamma$. Then, $E_M$ is the grounded kernel of some credulous extension $E$ of $\Gamma$.*

*Proof.* Let $\sigma$ be a path in $\Gamma$ beginning from some individual a. We prove that (i) if $\sigma \in E_M$ then $\sigma$ is defeasibly inheritable in $E_M$ and (ii) if $\sigma \notin E_M$ then $\sigma$ is not defeasibly inheritable in $E_M$. Like in other theorems we will show only the proof for the case $\sigma = \pi(a, \gamma, p)$ in $\Gamma$. We first prove (i). The constructibility of $\sigma$ is ensured by the definition of $E_M$. Suppose there exists a path $\alpha = \bar{\pi}(a, \beta, p)$ in $E_M$. Let $q \rightarrow_j p$ and $r \not\rightarrow_k p$ be the two last links of $\sigma$ and $\alpha$, respectively. $\alpha$ and $\sigma$ belong to $E_M$ mean that the two clauses $p(a) \leftarrow q(a)$ and $\neg p(a) \leftarrow r(a)$ belong to $P_\Gamma(M)$ which implies that *p(a)* and $\neg p(a)$ are contained in M. Contradictory ! because M is consistent. Hence, $\sigma$ is not-conflicted in $E_M$. Similarly, $\sigma$ is not preempted in $E_M$. So, $\sigma$ is defeasibly inheritance in $E_M$. Analogous, we can prove (ii). So, we have proved that $E_M = \{\sigma \mid \sigma$ begins from some individual a, $E_M \mid\sim \sigma\}$. Let $\Delta = \Gamma \setminus \{\text{j} \mid \text{j is a factual link in } \Gamma\}$. Clearly, there exists some credulous extension $E'$ of $\Delta$. We prove that $E = E' \cup E_M$ is a credulous extension of $\Gamma$. First, E is conflict free because none of the paths in E' is conflicted with some path in $E_M$. Further, for $\sigma \in E$ either $\sigma \in E'$ or $\sigma \in E_M$. It means $E' \mid\sim \sigma$ or $E_M \mid\sim \sigma$. Thus, $E \mid\sim \sigma$ for $\sigma \in E$ (*). Moreover, if $\sigma \notin E$ then either $\sigma$ is a path of $\Delta$ and so $E' \not\mid\sim \sigma$ or $\sigma$ begins from some individual node which implies that $E_M \not\mid\sim \sigma$. In both cases, it implies that $E \not\mid\sim \sigma$ (**). From (*) and (**) we can conclude that E is a credulous extension of $\Gamma$. The lemma is proved. □

The consequence of these two lemma is the theorem 13.

## Section 5

At first we state an important propositions of g-attacks.

**Lemma 35.** *Let $\Gamma$ be an acyclic, consistent inheritance network. Then, if $\sigma$ g-attacks $\tau$ and $\sigma$ is not a direct link then $deg_\Gamma(\sigma) < deg_\Gamma(\tau)$.*

*Proof.* By definition if $(\sigma, \tau) \in$g-attacks then $\tau$ is not a direct link. So, $\tau$ is a compound path. Hence, there is a prefix $\alpha = \pi(x, \eta, u) \rightarrow y$ (resp. $\pi(x, \eta, u) \not\rightarrow y$) of $\tau$ such that $\sigma = \pi(x, \delta, v, \theta, u)$ with $v \not\rightarrow y$ (resp. $v \rightarrow y$) in $\Gamma$ or $\sigma = \pi(x, \theta, v) \not\rightarrow y$ (resp. $\pi(x, \delta, v) \rightarrow y$) with $v \rightarrow u$ in $\Gamma$. By definition of $deg_\Gamma$ we have $deg_\Gamma(\sigma) < deg_\Gamma(\tau)$. The lemma is proved. □

In following we refer to a infinite sequence of arguments $A_1, A_2, \ldots, A_n, \ldots$ of an argumentation framework $AF$ with $A_{i+1}$ attacks $A_i$ for $1 \leq i$ as an infinite attack sequence. It follows directly from the property of $deg_\Gamma$ that

**Corollary 36.** *There exists no infinite g-attack sequence in $AF_0$=(AR, g-attacks).*

*Proof.* Suppose there is an infinite g-attack sequence $\sigma_1, \sigma_2, \ldots, \sigma_n, \ldots$ in $AF_0$. Since none of the direct links is g-attacked there is not a direct link in the sequence $\sigma_1, \sigma_2, \ldots, \sigma_n, \ldots$. By lemma 35 we have that $deg_\Gamma(\sigma_{i+1}) < deg_\Gamma(\sigma_i)$ for all i. It is contradictory to the assumption that AR is finite. □

It has been shown in [2] that if there exists no infinite g-attack sequence in an argumentation framework then grounded extension is also stable extension. Hence, we have:

**Lemma 37.** *For $AF_0$ the grounded extension $GE_0$ is a stable extension.*

*Proof.* Obvious □

**Lemma 38.** *If $\sigma$, $\tau$ are in conflict (i.e. $(\sigma, \tau) \in attacks_\Gamma$ but $(\sigma, \tau) \notin$ g-attacks) and $\sigma$, $\tau$ are in $GE_0$ then there is no argument $\delta$ in $GE_0$ such that $\sigma \in pre(\delta)$ and $\sigma \neq \delta$. (Likewise for $\tau$.)* □

Further, because $att{=}att^{-1}$ we can easily prove that $GE_1 = \{\sigma \in GE_0 \mid \nexists \tau \in GE_0 \; s.t. \; (\tau, \sigma) \in att\}$. We now prove that $GE_1$ coincides with HS. First, let $GE_1^i = \Gamma \cup \{\sigma \in GE_1 \mid deg_\Gamma(\sigma) \leq i\}$, e.g. $GE_1^i$ consists of all arguments in $GE_1$ which have the degree not greater than i. In following, we will write $F_{AF_1}$ or $F_{AF_0}$ to denote the characteristic functions of $AF_1$ and $AF_0$. We prove:

**Lemma 39.** *Let $\Gamma$ be an inheritance network and $GE_1$ be the grounded extension of $AF_1$. Then, $\Gamma \subseteq GE_1$.*

*Proof.* Because we consider only unambiguous network, there does not exist an argument $\sigma$ with $(\sigma, \tau) \in$g-attacks if $\tau$ is a direct link. Thus, $\Gamma \subseteq GE_0$. Further, there exists not $\sigma$ such that $(\sigma, \tau) \in att$ if $\tau$ is a direct link. Hence, $\Gamma \subseteq GE_1$. □

It results from lemma 39 the following proposition.

**Corollary 40.** $\Gamma{=}GE_1^1$

Further, we can prove:

**Lemma 41.** *Let $\sigma \in GE_1^{i+1}$. Then, $\sigma$ is permitted in $GE_1^i$.*

*Proof.* For a direct link the lemma is trivial. Now, let $\sigma = \pi(x, \tau, u) \rightarrow y \in GE_1^{i+1}$ (The proof is similar for negative path). We prove that $GE_1^i \mid\!\sim \sigma$ and $\sigma$ is permitted in $GE_1^i$. Since $GE_1^i \subseteq GE_1^{i+1}$ and there is no path conflicts with $\sigma$ in $GE_1^{i+1}$ we have that $\sigma$ is not conflicted in $GE_1^i$ (i). Further, for all prefix $\delta$ of $\sigma$ we have $\delta \in GE_1$ and $deg_\Gamma(\delta) < i{+}1$. Hence, $\delta \in GE_1^i$. So, $\sigma$ is constructible in $GE_1^i$ (ii). Since $\sigma \in GE_1^{i+1} \subseteq GE_1 \subseteq GE_0$ there is no argument in $GE_1^i$ that g-attacks $\sigma$. By definition of g- attacks we have $\sigma$ is not preempted in $GE_1^i$ (iii). It results from (i)-(iii) that $GE_1^i \mid\!\sim \sigma$ (iv). Now we prove that $\sigma$ is permitted in $GE_1^i$. It is clear, that $x \not\rightarrow y \notin \Gamma$ (v). We prove that there exists no argument $\tau$ conflicted with $\sigma$ such that $GE_1^i \mid\!\sim \tau$. Consider an arbitrary node v with $v \not\rightarrow y \in \Gamma$

such that there is some path $\delta = \pi(x, \rho, v)$ in $GE_1^i$. Let $\tau = \pi(x, \rho, v) \not\rightarrow y$. We prove that $GE_1^i \not\sim \tau$. From, $(\tau, \sigma) \in att$, $\sigma \in GE_1$ we have $\tau \notin GE_0$. ¿From lemma 37 $\exists \eta \in GE_0$, $\eta$ g-attacks $\tau$ but it cannot g-attacks $\delta$ since both $\eta$, $\delta$ belong to $GE_0$. Thus, $\eta$ g-attacks $\tau$ by a direct link or by preemption. It implies that $\eta = x \rightarrow y$ or $\eta = \pi(x, \alpha, z, \beta, v)$ with $z \rightarrow y \in \Gamma$. If $\eta = x \rightarrow y$ we have $\eta \in GE_1^i$ by corollary 40. Otherwise, we prove that there is no $\gamma$ in $GE_0$ such that $(\gamma, \eta) \in att$ by lemma 38 since if it is, $(\gamma, \delta) \in att$ and therefore $\delta \notin GE_1$. Contradictory !!! ¿From that we have $\eta \in GE_1$. In any case, $deg_\Gamma(\eta) < i + 1$. Hence, $\eta \in GE_1^i$. It implies that $GE_1^i \not\sim \tau$ (vi). From (iv)-(vi) we have that $\sigma$ is permitted in $GE_1^i$. □

**Lemma 42.** *$\Phi_i = GE_1^i$ for all i.*

*Proof.* We prove inductively over i. Corollary 40 makes the base case trivial. Assume that we have proved for i, that is, $\Phi_i = GE_1^i$. We now prove that $\Phi_{i+1} = GE_1^{i+1}$. Let $\sigma \in \Phi_{i+1}$. Since $\Phi_i \subseteq \Phi_{i+1}$, we need only to prove $\sigma \in GE_1^{i+1}$ for $\sigma \notin \Phi_i$. From the definition of the sequence $\Phi_i$, we can conclude that $deg_\Gamma(\sigma) = i + 1$. First, we prove that $\sigma \in GE_0$. If there exists no argument g-attacks $\sigma$ then clearly $\sigma \in F_{AF_1}(\emptyset)$ and hence, $\sigma \in GE_0$. Otherwise, assume there exists an argument $\alpha$ g-attacks $\sigma$. We now prove that $\alpha$ is g-attacked by $GE_0$. By definition of g-attacks there are three cases:

1. If $\alpha$ is a direct link, then $\sigma$ is preempted in $\Phi_i$. Contradictory !!! Hence, this case cannot occur.
2. $\alpha$ conflicts with a proper prefix of $\sigma$, say $\tau$. By inductive hypothesis, any proper prefix of $\sigma$ is contained in $GE_1^i$. So, $\tau \in GE_1^i$. Therefore, $\alpha \notin GE_0$. Hence, $\alpha$ is g-attacked by $GE_0$.
3. $\alpha$ g-attacks $\sigma$ by preemption. Without a lost of generality we can assume that $\sigma = \pi(x, \delta, u) \rightarrow y$, $\alpha = \pi(x, \tau, v, \eta, u)$ with $v \not\rightarrow y \in \Gamma$. Let $\beta = \pi(x, \delta, u)$. As $\sigma \in \Phi_{i+1}$ we have $\alpha \notin \Phi_i$. By inductive hypothesis we have $\alpha \notin GE_1^i$. $\alpha \notin GE_0$ because if $\alpha \in GE_0$ then $\exists \zeta \in GE_0$, $\zeta$ is conflicts with $\alpha$. But, $\zeta$ is conflicts with $\beta$ too, therefore $\beta \notin GE_1^i$. Contradictory !!! Hence, $\alpha \notin GE_0$. It results from lemma 37 that $\alpha$ is g-attacked by $GE_0$.

From this three cases we have that if $\alpha$ g-attacks $\sigma$, $\sigma \in \Phi_{i+1}$ then $\alpha$ is g-attacked by $GE_0$. Hence, $\sigma$ is acceptable wrt $GE_0$. So, $\sigma \in GE_0$. We now prove that there exists no argument $\tau$ in $GE_0$ such that $\tau$ is in conflict with $\sigma$. Assume the contrary, there is $\tau \in GE_0$ such that $\tau$ is conflict with $\sigma$. We prove that $\Phi_i \not\sim \tau$. Because of $deg_\Gamma(\sigma) = i + 1$, we have $deg_\Gamma(\tau) = i + 1$. Hence, $\tau$ is not conflicted in $\Phi_i$ (i). Let $\beta$ be the longest proper prefix of $\tau$. Hence, $deg_\Gamma(\beta) \leq i$. Clearly, $\beta \in GE_0$. Further, from lemma 38 there is no argument in conflict with $\beta \in GE_0$. Therefore, we have $\beta \in GE_1$. So, $\beta \in GE_1^i$ because $deg_\Gamma(\beta) \leq i$. By inductive hypothesis, $\beta \in \Phi_i$. Thus, $\tau$ is constructible in $\Phi_i$ (ii). Moreover, $\tau$ is not g-attacked by $GE_0$. It results that $\tau$ is not g-attacked by $\Phi_i$. That means, there is no direct link in $\Phi_i$ that attacks $\tau$ or $\tau$ is not preempted in $\Phi_i$ (iii). From (i)-(iii) we have that $\Phi_i \not\sim \tau$. But, then $\sigma$ and $\tau$ are conflict and both are defeasiby inheritable in $\Phi_i$. Thus, $\sigma \notin \Phi_{i+1}$. Contradictory !!!

We have proved that $\sigma \in GE_0$ and there is no argument $\tau \in GE_0$ such that $\tau$ is in conflict with $\sigma$. In other words, $\nexists\ \alpha \in GE_0$ with $(\alpha, \sigma) \in att$. Hence, $\sigma \in GE_1$. Because of $deg_\Gamma(\sigma) = i + 1$ we have $\sigma \in GE_1^{i+1}$ (*).

So, we have proved that $\Phi_{i+1} \subseteq GE_1^{i+1}$. We now prove that any argument $\sigma \in GE_1^{i+1}$ is contained in $\Phi_{i+1}$. This is done by lemma 41 because of any argument $\sigma \in GE_1^{i+1}$ is permitted in $GE_1^i = \Phi_i$. Hence, $\sigma \in \Phi_{i+1}$. (**)

The lemma is proved from (*) and (**). □

The consequence of this lemma is the theorem 23.

Further, we can easily prove the next lemma.

**Lemma 43.** *If $\sigma \in GE_{AF_\Gamma}$ then there exists no argument $\tau \in GE_0$ such that $(\sigma, \tau) \in att$.*

*Proof.* Let $\sigma$ be an arbitrary argument in $GE_{AF_\Gamma}$. ¿From the construction of $GE_{AF_\Gamma}$ we can assume that there is an integer i such that $\sigma \in F_{AF}^i(\oslash)$. We prove inductively over i that there is no $\tau$ in $GE_0$, $\tau$ attacks $\sigma$ by $att$. The base case (i=0) is trivial because there exists no argument $\tau$, $\tau$ attacks $\sigma$. Assume that for all j, $j \leq i$, the lemma is proved. We have to prove the inductive case. Assume that for some $\sigma \in F_{AF}^{i+1}(\oslash)$ there exists an argument $\tau \in GE_0$ such that $(\sigma, \tau) \in att$. Because $GE_{AF_\Gamma} \subseteq GE_0$ (section 5) both $\sigma$ and $\tau$ are contained in $GE_0$. Since $\tau$ attacks $\sigma$, by definition of grounded extension, there exists a $\sigma_1 \in F_{AF}^j(\oslash)$ where $j < i + 1$ that attacks $\tau$. $\sigma_1$ cannot attacks $\tau$ by an attacks in g-attacks because both $\sigma_1$ and $\tau$ are in $GE_0$. Thus, $(\sigma_1, \tau) \in att$. Since, $j < i + 1$, $\sigma_1 \in F_{AF}^j(\oslash)$, $\tau \in GE_0$, $(\tau, \sigma_1) \in att$. It is contradicted to the inductive hypothesis. Thus, our assumption is not correct. The lemma is proved. □

The corollary of the above lemma together with the result of theorem 20 is the theorem 24.